

الرجاء قراءة التعليمات الآتية قبل البدء في حل الإمتحان

- جزء الجبر يبدأ الحل من ناحية اليسار.
- جزء التفاضل يبدأ الحل من ناحية اليمين.
- الرسومات المطلوبة في الامتحان لا تحل في صفحات الرسم البياني.

Solve the following questions [full mark 50]

1<sup>st</sup> Question [8 marks]

Given the two polynomials

$$N(x) = 2x^2 - 7x + 9$$

$$D(x) = x^3 - 3x^2 + 4$$

(a) Show that the polynomial function  $D(x)$  touches the positive  $x$ -axis. [3 marks]

(b) Decompose the fraction  $\frac{N(x)}{D(x)}$ . [5 marks]

2<sup>nd</sup> Question [12 marks]

(a) Find the inverse of the following matrix using Gauss-Jordan method (row operations) [4 marks]

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

(b) Solve the following linear system of equations using inverse matrix method.

$$x - y = -2$$

$$x + z = 0$$

$$y + 2z = 2$$

[4 marks]

(c) Using Gauss-Jordan method, find the solution of the following linear system of equations. [4 marks]

$$2x + 5y + 5z = 0$$

$$y + z = 2$$

$$x + y + z = 1$$

من فضلك إقلب الصفحة باقى الأسئلة فى الخلف

**3<sup>rd</sup> Question [18 marks]**

(a) Evaluate  $\frac{dy}{dx}$  if  $y^3 = (\tan^{-1} x)^{\sec^{-1} x}$ . [5 marks]

(b) For the function  $f(x) = \left| \sin\left(\frac{x}{2}\right) \right|$ . [5 marks]

(i) Sketch the function  $f(x)$ .

(ii) From the graph of the above function, deduce domain, range, zeros, period and type of symmetry (odd or even).

(iii) Sketch the function  $\frac{1}{f(x)}$ .

(c) Determine the value of the  $\lim_{x \rightarrow 0^+} (1 + 3\sinh x)^{\csc x}$ . [4 marks]

(d) If  $y = (\sin^{-1} x)^3$  prove that  $(1 - x^2) y'' - x y' = 6 (\sqrt[3]{y})$ . [4 marks]

**4<sup>th</sup> Question [15 marks]**

(a) Prove that  $\frac{d}{dx} (\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$ . [3 marks]

(b) Complete the following [3 marks]

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(i)  $\lim_{x \rightarrow 0^+} (\sin x)^{1/x} = \dots\dots\dots$

(ii)  $\lim_{x \rightarrow \infty} \operatorname{sech}(e^x) = \dots\dots\dots$

(iii) The asymptotes of  $y = \tanh x$  are  $\dots\dots\dots$

(iv)  $\cosh x - \sinh x = \dots\dots\dots$

(v)  $\sec^{-1}(2) = \dots\dots\dots$

(vi)  $\cos(\cos^{-1} x) = x$  if  $x \in \dots\dots\dots$

(c) Evaluate  $\frac{dy}{dx}$  if  $y = e^{\tan^3(\sqrt{x^3 + 3^x + \ln 2})}$ . [3 marks]

(d) Simplify the expression  $e^{-\ln[\sec(2 \sin^{-1} x)]}$ . [3 marks]

(e) Find the Taylor expansion of the function  $f(x) = e^{3x-3}$  about  $x = 1$ . [3 marks]

With our best wishes  
Dr. Ayman Gomaa & Dr. Reda Abdo