تفاضل وتكامل متعدد المتغيرات

Mansoura University
Faculty of Engineering

Final Exam.
Saturday 23/12/2017

BCE
Building & Construction Engineering

Building & Construction Engineering Program

Prof. Dr. Magdi S. El-Azab | **Mathematics 4 (MATH 107)** | **Time allowed: 2 hours**

Books are allowed

Answer the following questions (Full mark 50 pts).

Question 1 [25 marks]

(a) [6 pts] Classify the points $x = 0$ and $x = 1$ for the differential equation

$$y'' + 4x^2y' - 12xy = 0,$$

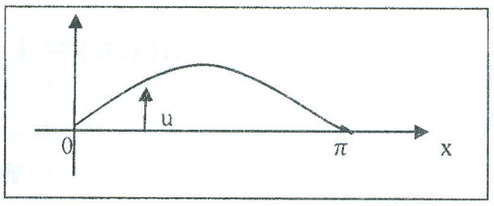
and then solve it in series about $x = 0$.

(b) [6 pts] Prove that for $0 \le x \le \pi$

$$x(\pi - x) = \frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{\cos 2nx}{n^2}$$

Sketch the graph of the function in the range $-2\pi \le x \le 2\pi$.

(c) [6 pts.] Derive a mathematical model for the transverse vibrations of a string with fixed ends (see the figure).



(d) [7 pts.] Use the technique of separation of variables to solve the following boundary value problem:

$$u_{tt} = 4u_{xx}, \qquad 0 \le x \le \pi, \quad t > 0,$$
$$u(0,t) = u(\pi,t) = 0, \qquad t > 0$$
$$u(x,0) = x(\pi - x), \qquad 0 \le x \le \pi$$
$$u_t(x,0) = x, \qquad 0 \le x \le \pi$$

Question 2 [25 marks]

(a) [6 pts] Find the Fourier integral representation of the function:

$$f(x) = \begin{cases} \cos x, & 0 \le x \le \pi \\ 0, & otherwise \end{cases}$$

(b) [6 pts] Using the graph in the xy-plane, classify the following partial differential equation

$$yu_{xx} + 2xu_{xy} - yu_{yy} + 4u_y - 6xy = 0$$

(c) [6 pts.] Build a model that describes the temperature distribution in a rod of length π made of homogeneous metal with constant cross section A that is completely insulated along its lateral edges.

(d) [7 pts.] Use the technique of separation of variables to solve the following boundary value problem:

$$u_t = 4u_{xx}, \qquad 0 \le x \le \pi, \quad t \ge 0,$$
$$u_x(0,t) = u_x(\pi,t) = 0, \qquad t \ge 0$$
$$u(x,\pi) = 1 - \frac{x}{\pi}, \qquad 0 \le x \le \pi$$

With all best wishes