



Answer the following questions (Full mark 50 pts.)

[All calculations must done correct to 4 decimal places]

يجب التأكد من كتابة القوانين جيدا مع العدادات الخاصة بكل قانون.

1. (a) [8 pts.] Drive the formula of the **False Position** iteration method, then use it to obtain the smallest positive root of the equation $f(x) = 0$, 4 iterations are required.

$$f(x) = 12\cos(x) - 24\sin(x) + 5.$$

- (b) [6 pts.] Use the table shown to fit the equation $y = ae^{bx}$

x	0.5	1	2	3	4
y	10.4	5.6	3.2	2.4	2.1

2. (a) [8 pts.] Given the following table,

x	1	2	3	4	5
$y = f(x)$	1	8	25	60	120

By using two different Newton's formulas find $f(1.25)$.

- (b) [8 pts.] Given the initial value problem $dy/dx = -y + x + 1$, $y(0) = 0.5$. By using the third order **Taylor** series, find $y(0.1)$. Also by using **Runge-Kutta** method of order 4, find $y(-0.1)$.

- (C) [6 pts.] Use **Composite Simpson's 1/3 rule** to approximate the following integration:

$$\int_0^{1.2} \sqrt{\sin x + \cos x} dx.$$

Solve by taking $[n = 6]$.

3. [4 pts.] For the parabolic partial differential equation,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

Drive the explicit equation used to approximate the solution by finite difference method.

4. [12 pts.] Find the approximate solution to the boundary value problem,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10x^2y,$$

$$0 \leq x \leq 1.5, \quad 0 \leq y \leq 2.$$

With the boundary conditions:

$$u(0, y) = u(1.5, y) = 20y(2 - y), \quad 0 \leq y \leq 2$$

$$u(x, 0) = u(x, 2) = 15x$$

Draw the solution mesh and show the grid points and boundary values.

By taking $(n = 3, m = 4)$,

Then use finite difference method to

- 1) Describe the resulting system. (*P. S.* : find the system of equations), then
- 2) Solve the resulting system by Gauss-Seidel method. (5 iterations are required).