Mansoura University Building & Construction Engineering	First Semester Examination Academic Year 2012-2013 Math 1- Calculus 1 BCE students – Level (000)	Date : 12 Jan. 2013 Duration : 2 Hours Paper Mark : 56 Counted Mark : 50

This paper contains FOUR questions which carry equal marks

Approximate mark weighting for each section is indicated in a square bracket, e.g. [3]

Make sure that the paper is two-sided (including 4 different questions)

Question 1 [14 marks]

For the function $f(x) = 1+\dfrac{3}{x^2-4}$

1) Determine the domain [1]
2) Discuss the *symmetry* (whether even, odd, or neither) [1]
3) Evaluate $\lim\limits_{x\to 2^-} f(x)$, $\lim\limits_{x\to 2^+} f(x)$, and $\lim\limits_{x\to\infty} f(x)$ [2]
4) Find the points of *intersection* with the coordinate axes [2]
5) Find the horizontal and vertical *asymptotes,* if exist. [2]
6) Find the equations of *tangent* line and *normal* line at $(1,0)$ [3]
7) Graph the function [3]

Question 2 [14 marks]

a) Find $\dfrac{dy}{dx}$ if $(x+1)y+\ln\left(\dfrac{e^{2y}}{x+1}\right)^{x+1}=1$ [4]

b) Given that $y=(x+1)^{\sin x}$, $x=t^2+3\cosh t$, determine $\dfrac{dy}{dx}$, $\dfrac{dx}{dt}$, then use the chain rue to find $\dfrac{dy}{dt}$ [4]

c) The function $y(x)$ satisfies [6]

$$y''+(y')^2-3y=x^2+1, \qquad y(0)=1, \quad y'(0)=2$$

Write a *third order* Taylor polynomial for $y(x)$ near $x=0$, then estimate $y(0.1)$.

Please turn over

Question 3 [14 marks]

a) For the function $f(x) = 2x^3 - 3x^2 - 12x + 1$, find the local and absolute maximum and minimum on the interval $[-2, 3]$. **[4]**

b) Evaluate the following limits

1) $\lim_{x \to 0} \left(\dfrac{\tan 2x}{x + 1 - \cos(2x)} \right)$ **[3]**

2) $\lim_{x \to \infty} \left(e^{2x} + 2 \right)^{(1/x)}$ **[3]**

c) Use partial fraction decomposition to factorise $\dfrac{2x}{(x-1)(x^2+1)}$ **[4]**

Question 4 [14 marks]

a) Given the matrices

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 2 \\ -3 & 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

i) Evaluate $\det(A^T)$ and $2AC$ **[2]**

ii) Prove that A is the inverse matrix of B **[2]**

iii) Determine X and Y which satisfy the two linear algebraic systems

$$B^{-1}X = 2AC \qquad \text{and} \qquad BY + 2C = X$$ **[4]**

b) For the linear algebraic system

$$\begin{aligned} x + y - z &= 3 \\ x + 2y + z &= 3 \\ x + y + (\alpha^2 - 17)\, z &= \alpha \end{aligned}$$

Specify the possible values of the parameter α (if exist) such that the system has

1) No solution **[2]**
2) Infinite number of solutions (then find one of them) **[2]**
3) Unique solution, then find it. **[2]**

-- End of Exam ----------------