Mansoura University
Faculty of Engineering
Building & Construction Eng. Program
Course Title: Differential Equations
Course Code: MATH106

Final Exam (50%)
Time: 2 hours
Date: January 21th 2017
Lecturer: Dr. Mohamed Soror

(١٠٠) معادلات تفاضلية

Question 1 (10 Marks)

Solve the following differential equations

i) $ydx = x(\ln y - \ln x + 1)dy,$ [3 Marks]

ii) $(x^2 - \sin^2 y)dx + x \sin 2y \; dy = 0,$ [3 Marks]

iii) $y'' + 4y = 4 \sin 2x.$ [4 Marks]

Question 2 (8 Marks)

i) Find the Laplace transform of $f(t) = t\, e^t \sin t \cos t \cosh t,$ [4 Marks]

ii) Find the inverse Laplace transform of $F(s) = \frac{s}{(s+1)(s^2+4s+5)}.$ [4 Marks]

Question 3 (8 Marks)

Solve the following system of differential equations

$$f_1'(t) = 3f_1(t) - 2f_2(t), \quad f_2'(t) = 2f_1(t) - 2f_2(t), \; f_1(0) = -1, \; f_2(0) = 1.$$

Question 4 (8 Marks)

If $z = \sin^{-1}\left(\frac{y^2-xy}{x-y}\right)$. Show that

$$xz_x + yz_y = \tan z, \qquad x^2z_{xx} + 2xyz_{xy} + y^2z_{yy} = \tan^3 z.$$

Question 5 (8 Marks)

i) A rectangular box without a lid is to be made from $18\, m^2$ of cardboard. Find the maximum volume of such a box. [5 Marks]

ii) Find the directions in which the directional derivative of $f(x,y) = x^2 + \sin xy$ at the point (1,0) has the value 1. [3 Marks]

Question 6 (8 Grades)

Evaluate the following double integrals

i) $\iint_D (x+y)\, dA,$ D is bounded by $y = \sqrt{x}$ and $y = x^2,$ [4 Marks]

ii) $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dxdy.$ [4 Marks]