



Answer All Questions

Questions No.[1](20 Marks)

(a) If $a > 0$ and b is independent of a and

$$\int_0^b \frac{dx}{a^2 + x^2}$$

and hence evaluate

$$\int_0^\infty \frac{dx}{(a^2 + x^2)^2} \quad [5 \text{ Marks}]$$

(b) If

$$z = \ln \frac{x^5 + x^2 y^3 + x^3 y^2 + y^5}{x^3 + x^2 y + y^3}$$

Prove that

$$x z_x + y z_y = 2 \quad [5 \text{ Marks}]$$

(c) Find the shortest distance from the point $(1, 0, -2)$ to the plane

$$x + 2y + z = 4 \quad [5 \text{ Marks}]$$

(d) Evaluate

$$\iint_D (3x + 4y^2) dA$$

where D is the region in the upper half-plane bounded by the circles

$$x^2 + y^2 = 1 \quad \text{and} \quad x^2 + y^2 = 4 \quad [5 \text{ Marks}]$$

Questions No.[2](20 Marks)

(a) Solve by any method

$$1. \quad 3x^2 y'' + 11xy' - 3y = 0 \quad [5 \text{ Marks}]$$

$$1. \quad y'' + (3xy - 1)y' = 0 \quad [5 \text{ Marks}]$$

(b) Determine the proper form of $y_p(x)$ for

$$y'' + 2y' + y = 3e^{-x}$$

but do not solve for the undetermined coefficient

Questions No.[3](15 Marks)

(a) Find the Laplace transform of the function

$$f_1(t) = \begin{cases} \sin t, & 0 < t < \pi, \\ 0, & \pi < t < 2\pi. \end{cases} \quad [4 \text{ Marks}]$$

(b) Find the inverse Laplace transform of

$$F_1(s) = \left\{ \ln \frac{s+3}{s+4} \right\}, \quad F_2(s) = \frac{s+1}{s^2(s+2)^3}. \quad [6 \text{ Marks}]$$

(c) Evaluate

$$\int_0^{\infty} \frac{e^{-3t} - e^{-6t}}{t} dt. \quad [5 \text{ Marks}]$$