



**Answer All Questions**

**Question 1 [25 Marks]**

- (a) Evaluate the following integrals: (i)  $\int x \tan^{-1} x \, dx$  (ii)  $\int \frac{\sin^{-1} \sqrt{x}}{\sqrt{x-x^2}} \, dx$  [7 marks]
- (b) Find a reduction formula to  $I_n = \int \sinh^n x \, dx$  and then evaluate the integral  $\int \sinh^5 x \, dx$  [6 marks]
- (c) Find the area bounded by the parabola  $y = 9 - x^2$  and the  $x$ -axis. If this area is revolved about  $y$ -axis, find the volume of the solid of revolution. [6 marks]
- (d) Test the improper integral  $I = \int_0^{\infty} \frac{1}{e^{x^2} + 7} \, dx$  for convergence or divergence. [3 marks]
- (e) Test the infinite series  $\sum_{n=1}^{\infty} \frac{5^n}{n}$  for convergence or divergence. [3 marks]

**Question 2 [25 Marks]**

- (a) Find the equation of the circle whose center is the point of intersection of the two straight lines:  $y^2 + xy - 4y = 0$  and passing through the origin. Find also the angle between the two lines. [4 marks]
- (b) Find the length major and minor axes of the ellipse:  $2x^2 + y^2 - 4y = 0$ . Sketch the ellipse and find its eccentricity. [4 marks]
- (c) Find the equation of rectangular hyperbola whose focus is the origin and directrix is the line:  $x + y - 2 = 0$ . Find the conjugate hyperbola and their common asymptotes. [6 marks]
- (d) Find the equation of line perpendicular to the plane  $2x + 4y - 3z = 12$  and passing through the point  $(2, 0, -1)$ . Find the point of intersection of this line with the  $xy$ -plane. [5 marks]
- (e) Find the equation of the main axis of the cone  $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{2} - \frac{(z-2)^2}{9} = 0$ . Find also the intersection with the planes: (i)  $x = 1$  (ii)  $z = 2$ . [6 marks]