تفاضل وتكاملات

Mansoura University
Faculty of Engineering

Final Exam.
Saturday 5/9/2015

BCE
Building & Construction Engineering

Building & Construction Engineering Program

Prof. Dr. Magdi S. El-Azab | Mathematics 2 | Time allowed: 2 Hours

Answer the following problems [Full mark 50 pts.]

1. *(a)* **[6 pts.]** Evaluate each of the following integrals

(*i*) $I_1 = \int_{-\pi/2}^{\pi/2} (\cos^6 x - x^2 \sin^3 x)\, dx$ (*ii*) $I_2 = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

(*iii*) $I_3 = \int_1^e x^2 (lnx)^2\, dx$ (*iv*) $I_4 = \int (\sec x\, e^{tan})^2\, dx$

(*v*) $I_5 = \int \cos 4x \cos 2x\, dx$ (*vi*) $I_6 = \int \frac{8x-1}{(x-2)(x+3)} dx$

(b) **[6 pts.]** If

$$y'' = 4\pi(\cos 2x - \sin 2x),$$

find y' and y given that $y = \pi^2,\ y' = 0$, when $x = \frac{\pi}{2}$.

(c) **[6 pts.]** If $f(x) = \int_x^{x^2} \frac{\cosh(t^2)}{e^t} dt$, *find* $\frac{df}{dx}$.

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2. *(a)* **[6 marks]** Find a reduction formula for the following integral

$$I_n = \int (\sin^{-1} x)^n\, dx,$$

and hence evaluate I_3.

(b) **[6 marks]** Evaluate the upper and lower values of the integral $\int_0^{2\pi} \frac{dx}{\sqrt{16 + 9\sin^2 x}}$.

(c) **[6 pts.]** Test for convergence or divergence each of the following series

$$(i)\ \sum_{n=1}^{\infty} \frac{n}{n^2+3} \qquad (ii)\ \sum_{n=1}^{\infty} \frac{n^3}{4^n} \qquad (iii)\ \sum_{n=1}^{\infty} \frac{1}{2^n}\left(1+\frac{1}{n}\right)^{n^2}$$

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3. (a) **[6 pts.]** Given the function $f(x) = \tan x$, defined on the interval $\left[0, \frac{\pi}{4}\right]$. Show that this function satisfies the hypotheses of the mean value theorem for differentiation and integration, and then find all the points c satisfying the conclusion of each theorem.

(b) **[6 pts.]** Find the area of the region bounded by the curves

$$y = 4 + \sin^3 x, \quad x = 0, \quad x = \frac{\pi}{2}, \quad x - axis.$$

If this area is rotated about the x-axis, find the volume of the generated solid.

(c) **[6 pts.]** Find the arc length of the curve $y = x^2$ from $x = 0$ to $x = 1$. If this curve rotates about the x-axis, find the area of the generated surface.

With all best wishes