



Answer the following questions:

Question (1) [15 marks]

- a) [7 marks] For the equation  $x^2 - 2x - 4y + 9 = 0$ . Find vertex, focus, axis, directrix, length of LR and ends of LR.
- b) [4 marks] Find the equation of the plane containing z-axis and perpendicular to the plane  $x - 2y + z + 1 = 0$ .
- c) [4 marks] Find the equations of the line of intersection of the two planes:  
 $x + y - 2z = 2$  and  $2x - y - z = 1$

Question (2) [10 marks]

- a) [4 marks] For the sphere  $x^2 + y^2 + z^2 - 2z + c = 0$ . If the plane  $x + 2y + 2z + 4 = 0$  touches this sphere, find the value of the constant  $c$ .
- b) [6 marks] Given the surface equation  $x^2 + 2z^2 = (y - 1)^2$
- State the surface name, its vertex and its main axis.
  - Describe the traces in the  $xy$ -plane, the plane  $z = 1$  and the plane  $y = 1$ .

Go to the next page 

**Question (3) [9 marks]**

[3 marks each] Evaluate the following integrals (if it exists):

a)  $\int \frac{x^2}{(1-x^2)^{3/2}} dx$

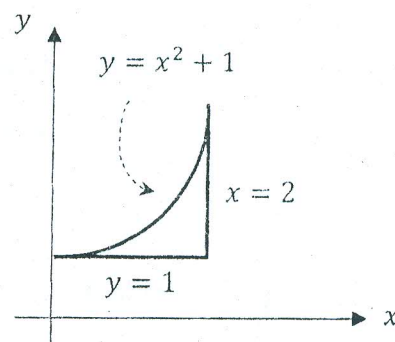
b)  $\int_{-1}^1 \frac{1 + \tan^5 x}{1 + x^2} dx$

c)  $\int_0^{\infty} \frac{1}{x^2} dx$

**Question (4) [16 marks]**

a) [4 marks] Find the area of the region bounded by  $y = \sin x$  and the  $x$ -axis on the interval  $[0, \frac{3\pi}{2}]$ .

b) [6 marks] Find the volume of the solid generated by revolving the bounded region shown in the figure:



- i. about the line  $y = 1$ .
- ii. about the  $y$ -axis.

c) [6 marks]

i. Find the length of the arc

$$x = 2 + 3t, \quad y = \cosh 3t, \quad 0 \leq t \leq 1.$$

ii. Find the area of the surface generated by rotating this arc about the line  $x = 2$ .

**Best Regards, Dr. Mustafa El-Agamy**