رياضة ١ (000)

Mansoura University
Faculty of Engineering
BCE Program
Course Title: Mathematics(1)
Course Code: MATH001

First Semester 2016/2017
Level 000 students
Final Exam (50%)

Allowed Time: 2 hours
Exam in two sided paper
Date: Sat, 21 / 1 / 2017
Lecturer:
Dr. Samir Shamseldeen

Answer ALL the following questions

Question 1 [12 marks]

(a) Use partial fractions to decompose [5]

$$\frac{x-1}{x^4+x^3+x^2+x}$$

(b) Solve the following system of linear equations using Gauss-Jordan method [4]

$$x_1 - 3x_2 + 4x_3 = 12$$
$$2x_1 - 5x_2 + 3x_3 = 13$$
$$x_1 - 4x_2 + 9x_3 = 23$$

(c) Solve the following equation if its roots form an arithmetic sequence [3]

$$x^3 - 15x^2 + 59x - 45 = 0$$

Question 2 [13 marks]

(a) Find $\frac{dy}{dx}$ if [4]

$$(\sin^{-1} x)^y = x\, e^{\cosh y}.$$

(b) Find Taylor series of order two near $x_0 = 1$ for $f(x) = \tan^{-1} x$ [3]

(c) Evaluate the following limits

(i) $L_1 = \lim_{x\to 0} (\csc x - \cot x)$ [2]

(ii) $L_2 = \lim_{x\to 0^+} \left(e^{\sin x} \cosh x\right)^{\csc x}$ [4]

Question 3 [12 marks, 2 for each part]

(a) Determine the zeroes of the function $f(x) = \tan(x/4)$ in the interval $[-\pi, 5\pi]$

(b) Solve the equation $ln(cosh(x) + sinh(x)) = x^3 - 3x$

(c) Find the equation of the tangent line at $x = 0$ to the curve $x\,cos(\pi y) - e^x - y = 0$

(d) Graph the periodic function $f(x) = 1 - \sin(2x)$ in two complete successive periods

(e) Given the function $g(x) = 1 + \cosh(x)$. Graph the reciprocal function $1/g(x)$

(f) Given the function $f(x) = 1 + e^x$. On the same axes, graph f and its inverse f^{-1}

Question 4 [16 marks]

(a) Find the possible values of the two constants p and q if the following system of equations is inconsistent (has no solution) [4]

$$x_1 - 2x_2 + 2x_3 = 1, \quad x_1 - x_2 + x_3 = 2, \quad -x_1 + 2x_2 + p\,x_3 = q$$

(b) Given the matrices [12]

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

1) **True or False**: (Justify your answer, **no credit for no explanations**)

   (i) The multiplication $C^T B A^T$ is **not** defined

   (ii) If $G = A + B$, then G is a singular 3×3 matrix

   (iii) The inverse of B is the matrix A

   (iv) $\det(BAB^T) = \det(A)$.

2) Find the matrix X if $A^{-1}X = BA^TC$.

End of Exam

Best of Luck, *Dr. Samir Shams* 21/1/2017