Mansoura University Faculty of Engineering	BCE Level100 First Term Exam	
Math. & Engineering Physics Dept. Instructor: Prof. Dr. Mohamed El-Gamel	Math. (106) Full Mark: 50	Time : 2 hours Final Exam: 25-12- 2017

(1)-[a] Find the maximum and minimum values of [4 pts]

$$f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$$

[b] Let

$$f(x, y, z) = e^{x^2 y^3 z^4}$$ [4 pts]

and $\quad F = \left(x^2 z - yz\right) i + z\, e^{2y} j + y e^{z-x} k$

Decide which of the following operations are defined and evaluate those that are defined

$$\nabla f, \quad \nabla \cdot f, \quad \nabla F, \quad \nabla \cdot (\nabla \times \nabla f)$$

[c] Find [4 pts]

$$\frac{d}{d\alpha} \int_{-2\alpha^2}^{-\alpha} e^{\alpha x^3} dx$$

[d] Evaluate the following integrals [12 pts]

$$\int_0^1 \int_{\sqrt{y}}^1 \cos x^3 \, dx \, dy \qquad \int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{x^2+y^2} \, dx \, dy$$

$$\int_1^2 \int_x^{x^2} \int_0^{\ln x} x\, e^y \, dy \, dz \, dx$$

(2)-[a] Solve by any method [20 pts]

1. $y''' - 6y'' + 12y' - 8y = 17 e^{2x}$

2. $y + \dfrac{xy'}{2} - \dfrac{2\sqrt{y}}{x} = x^4 \sqrt{y}\, e^{-x\sqrt{y}}$

3. $\sin^{-1}\left(\dfrac{dy}{dx}\right) = x + y$

4. $x^2 y'' - 2x\, y' + 2y = \cos\left(\dfrac{1}{x}\right)$

(b) Determine whether the functions y_1 and y_2 are linearly dependent or independent [4 pts]

$$y_1 = \ln x \qquad y_2 = \ln x^n$$

(c) Find a second linearly independent solution. What is the general solution of the differential equation? [4 pts]

$$y'' - y = 0 \qquad\qquad y_1(x) = e^x$$