Mansoura University
Faculty of Engineering

Final Term Exam.
Sunday, 13/1/2013

BCE
Building & Construction Engineering

Building & Construction Engineering Program

Prof. Dr. Magdi S. El-Azab | **Math 106 (Differential equations)** | **Time allowed: 2 Hours**

Answer the following problems (Full mark 50 pts)

1. **(a)** [5 pts] If $u = y^{2x}$, find the values of α to satisfy $\frac{1}{\ln y} u_x + \frac{y}{x} u_y = \alpha^2 u$.

(b) Given the two vectors: $\vec{F} = y^2 z^3\,\mathbf{i} + ax\,yz^3\,\mathbf{j} + bx\,y^2 z^2\,\mathbf{k}$.

(i) [5 pts] If $\vec{F}$ is an irrotational vector field. Compute the constants a and b and then find its potential function $\varphi(x, y, z)$.

(ii) [5 pts] Find *divgrad* φ, and $\nabla \times (\nabla\varphi)$.

(iii) [5 pts] Find the equation of the tangent plane at the point $(-2, 1, -3)$ to $\varphi(x, y, z) = 3$.

(c) [5 pts] Test for local maxima and minima for the function

$$f(x, y) = x^3 - 2xy + y^2 - x$$

2. **(a)** [5 pts] Find the volume of region R bounded by the solid that lies above the cone $z = \sqrt{3(x^2 + y^2)}$ and below the sphere $x^2 + y^2 + z^2 = 4z$. Then use the divergence theorem to compute $\iint_S F \cdot ds$ given that

$$F = (x + \cos y)\mathbf{i} + (y + \sin z)\mathbf{j} + (z + e^x)\mathbf{k},$$

and S is the surface of the region R.

(b) [5 pts] Verify Stokes' theorem for the vector field $\vec{F} = -y\mathbf{i} - z\mathbf{j} - x\mathbf{k}$ and S is the paraboloid $z = x^2 + 4y^2$, $z \le 4$.

3. (a) Solve the following differential equations

(i) [3 pts] $y' = \frac{y}{x} + x\sin\left(\frac{y}{x}\right)$

(ii) [3 pts] $(\sec x - y)\,dx - (x - \ln y - 2)\,dy = 0$

(iii) [4 pts] $(D^4 - 2D^3 + 5D^2)y = 30\sin x + 20e^{-3x} + 10$

(b) [5 pts] Evaluate the function $u(x)$ which satisfies the relation $\frac{d}{dx}(u(x)v(x)) = \frac{du}{dx}\cdot\frac{dv}{dx}$ if $v(x) = e^{x^2}$.