



رقم الفصل:

اسم الطالب:

الامتحان في أربع صفحات مسموح الحل بالقلم الرصاص معطي جدول المشتقات في الصفحة الثانية

Answer the following Problems

Problem 1 [6 marks]

Complete the following table (using the given functions)

$f(x) = \tanh^{-1} x$	$f(x) = \ln x$
Domain	
Range	
Zeros	
Asymptotes	
$\lim_{x \rightarrow 1^-} (\tanh^{-1} x) =$	$\lim_{x \rightarrow 0^+} (\ln x) =$

(1/4)

Problem 2 [4 marks]

Find $\frac{dy}{dx}$ of the following functions:

i) $y = (e^{\sin^{-1} x}) (\sec(x^3 \ln x))$

Solution

ii) $y = \sinh(\tanh^{-1}(3^{\tan^{-1} x}))$

Solution

$f(x)$	x^n	\sqrt{x}	e^x	a^x	$\ln x$	$\sec x$	$\sin x$
$f'(x)$	nx^{n-1}	$1/2\sqrt{x}$	e^x	$a^x \ln a$	$1/x$	$\sec x \tan x$	$\cos x$

$f(x)$	$\cosh x$	$\sinh x$	$\tanh x$	$\tanh^{-1} x$	$\tan^{-1} x$	$\sinh^{-1} x$	$\sin^{-1} x$
$f'(x)$	$\sinh x$	$\cosh x$	$\text{sech}^2 x$	$\frac{1}{1-x^2}$	$\frac{1}{1+x^2}$	$\frac{1}{\sqrt{1+x^2}}$	$\frac{1}{\sqrt{1-x^2}}$

(2/4)

Problem 3 [6 marks]

Find $\frac{dy}{dx}$ of the following:

i) $y = (4^x \tanh x)^{(\sinh^{-1}(\sqrt{x}))}$

Solution

ii) $\cosh(x + y) = y^2 \sin x$

Solution

(3/4)

Problem 4 [4 marks]

i) At what point on the curve $y = e^x$ is the slope of the tangent equal two?

Solution

ii) Prove that $\sinh^{-1} x = \ln(x + \sqrt{1 + x^2})$, $x \in \mathbb{R}$

Solution

(4/4)



رقم الفصل:

اسم الطالب:

معطي جدول للمشتقات في الصفحة الثانية

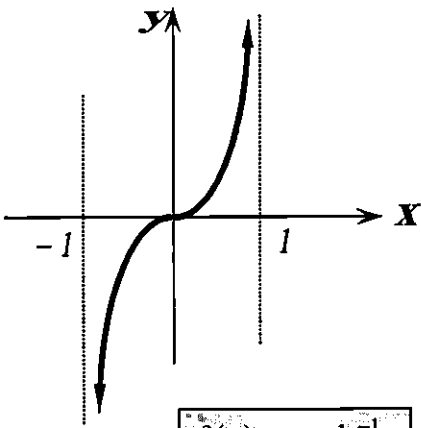
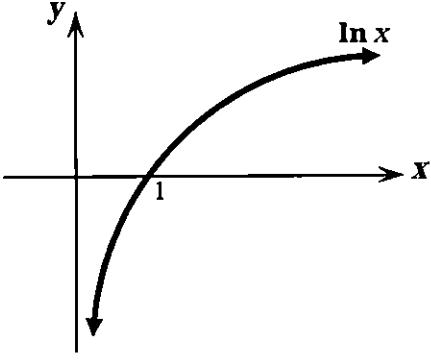
مسموح الحل بالقلم الرصاص

الامتحان في اربع صفحات

Answer the following Problems

Problem 1 [6 marks]

Complete the following table (using the given functions)

 <p>$f(x) = \tanh^{-1} x$</p>	 <p>$f(x) = \ln x$</p>
$] -1, 1[$	R^+
R	R
$x = 0$	$x = 1$
$x = \pm 1$	$x = 0$
$\lim_{x \rightarrow 1^-} (\tanh^{-1} x) = \infty$	$\lim_{x \rightarrow 0^+} (\ln x) = -\infty$

Problem 2 [4 marks]

Find $\frac{dy}{dx}$ of the following functions:

i) $y = (e^{\sin^{-1} x}) (\sec (x^3 \ln x))$

Solution

$$y' = (e^{\sin^{-1} x}) \left[\sec(x^3 \ln x) \tan(x^3 \ln x) \left(x^3 \left(\frac{1}{x} \right) + 3x^2 \ln x \right) \right] \quad \boxed{1}$$

$$+ (\sec (x^3 \ln x)) \left[e^{\sin^{-1} x} \frac{1}{\sqrt{1-x^2}} \right] \quad \boxed{1}$$

ii) $y = \sinh(\tanh^{-1}(3^{\tan^{-1} x}))$

Solution

$$y' = \cosh(\tanh^{-1}(3^{\tan^{-1} x})) \times \frac{1}{1 - (3^{\tan^{-1} x})^2} \times 3^{\tan^{-1} x} \ln 3 \times \frac{1}{1 + x^2}$$

$\boxed{1/2}$
 $\boxed{1/2}$
 $\boxed{1/2}$
 $\boxed{1/2}$

$f(x)$	x^n	\sqrt{x}	e^x	a^x	$\ln x$	$\sec x$	$\sin x$
$f'(x)$	nx^{n-1}	$1/2\sqrt{x}$	e^x	$a^x \ln a$	$1/x$	$\sec x \tan x$	$\cos x$

$f(x)$	$\cosh x$	$\sinh x$	$\tanh x$	$\tanh^{-1} x$	$\tan^{-1} x$	$\sinh^{-1} x$	$\sin^{-1} x$
$f'(x)$	$\sinh x$	$\cosh x$	$\operatorname{sech}^2 x$	$\frac{1}{1-x^2}$	$\frac{1}{1+x^2}$	$\frac{1}{\sqrt{1+x^2}}$	$\frac{1}{\sqrt{1-x^2}}$

Problem 3 [6 marks]

Find $\frac{dy}{dx}$ of the following:

i) $y = (4^x \tanh x)^{(\sinh^{-1}(\sqrt{x}))}$

Solution

$$y' = (4^x \tanh x)^{(\sinh^{-1}(\sqrt{x}))} \times (\sinh^{-1}(\sqrt{x}) \ln(4^x \tanh x))'$$

$$= (4^x \tanh x)^{(\sinh^{-1}(\sqrt{x}))} \times (\sinh^{-1}(\sqrt{x}) \times [x \ln 4 + \ln(\tanh x)])'$$

$y' = (4^x \tanh x)^{(\sinh^{-1}(\sqrt{x}))} \times$ $\left(\sinh^{-1}(\sqrt{x}) \left[\ln 4 + \frac{\operatorname{sech}^2 x}{\tanh x} \right] + [x \ln 4 + \ln(\tanh x)] \times \frac{1}{\sqrt{1+x}} \frac{1}{2\sqrt{x}} \right)$	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1/2</div>
<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px 5px;">1/2</div> <div style="border: 1px solid black; padding: 2px 5px;">1/2</div> <div style="border: 1px solid black; padding: 2px 5px;">1/2</div> </div>	<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px 5px;">1/2</div> <div style="border: 1px solid black; padding: 2px 5px;">1/2</div> </div>

Another Solution

$$\ln y = (\sinh^{-1}(\sqrt{x})) \ln(4^x \tanh x) = (\sinh^{-1}(\sqrt{x}) \times [x \ln 4 + \ln(\tanh x)])$$

$$\frac{y'}{y} = \sinh^{-1}(\sqrt{x}) \left[\ln 4 + \frac{\operatorname{sech}^2 x}{\tanh x} \right] + [x \ln 4 + \ln(\tanh x)] \times \frac{1}{\sqrt{1+x}} \frac{1}{2\sqrt{x}}$$

$$y' = y \left(\sinh^{-1}(\sqrt{x}) \left[\ln 4 + \frac{\operatorname{sech}^2 x}{\tanh x} \right] + [x \ln 4 + \ln(\tanh x)] \times \frac{1}{\sqrt{1+x}} \frac{1}{2\sqrt{x}} \right)$$

$$y' = (4^x \tanh x)^{(\sinh^{-1}(\sqrt{x}))} \times$$

$$\left(\sinh^{-1}(\sqrt{x}) \left[\ln 4 + \frac{\operatorname{sech}^2 x}{\tanh x} \right] + [x \ln 4 + \ln(\tanh x)] \times \frac{1}{\sqrt{1+x}} \frac{1}{2\sqrt{x}} \right)$$

ii) $\cosh(x + y) = y^2 \sin x$

Solution

$\sinh(x + y) \times (1 + y') = y^2 \cos x + 2yy' \sin x$	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">2</div>
$y' \sinh(x + y) + \sinh(x + y) = y^2 \cos x + 2yy' \sin x$	
$y' \sinh(x + y) - 2yy' \sin x = y^2 \cos x - \sinh(x + y)$	
$y' = [y^2 \cos x - \sinh(x + y)] / [\sinh(x + y) - 2y \sin x]$	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1</div>

Problem 4 [4 marks]

i) At what point on the curve $y = e^x$ is the slope of the tangent equal two?

Solution

$$y' = e^x = \text{slope of tangent at } (x, y) = 2$$

1

$$\Rightarrow e^x = 2 \Rightarrow x = \ln 2 \Rightarrow y = e^{\ln 2} = 2$$

$1/2 + 1/2$

Thus the point of contact is $(\ln 2, 2)$

ii) Prove that $\sinh^{-1} x = \ln(x + \sqrt{1 + x^2})$, $x \in R$

Solution

$$\text{Let } y = \sinh^{-1} x \Rightarrow \sinh y = x \Rightarrow e^y - e^{-y} = 2x$$

1

$$\Rightarrow (e^y)^2 - 1 = 2x(e^y) \Rightarrow (e^y)^2 - 2x(e^y) - 1 = 0$$

$$\Rightarrow e^y = \frac{2x + \sqrt{4x^2 + 4}}{2} \quad (-\text{rejected})$$

$1/2$

$$e^y = x + \sqrt{x^2 + 1} \Rightarrow y = \ln(x + \sqrt{x^2 + 1}) \Rightarrow \sinh^{-1} x$$

$1/2$

$$= \ln(x + \sqrt{x^2 + 1})$$