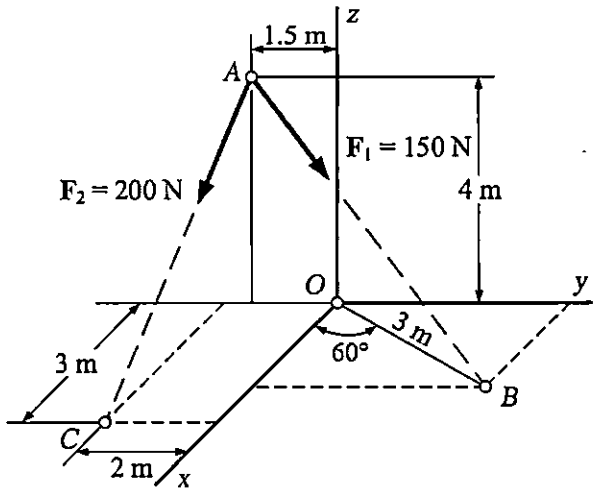
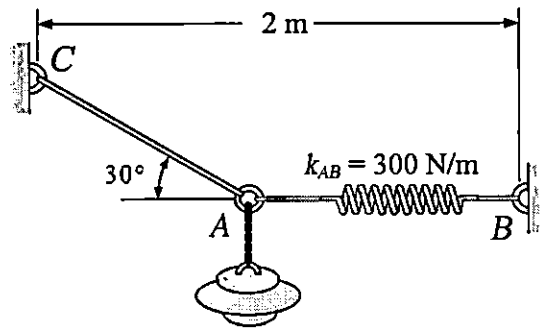
Mansoura University
Faculty of Engineering
MTE, BCE, BME, CIE Level: 000

Date: 14/11/2019
Mechanics 1
Mid Term Exam. Time: 1 hr

1] Determine the magnitude and coordinate direction angle of the resultant force acting at point *A*. [5 degrees]



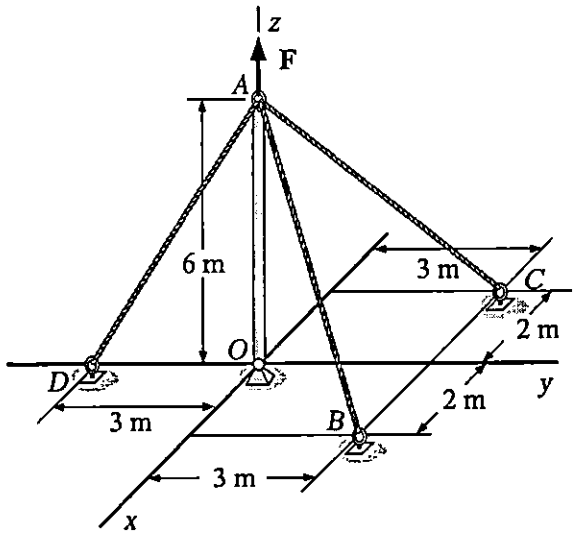
Prob. (1)



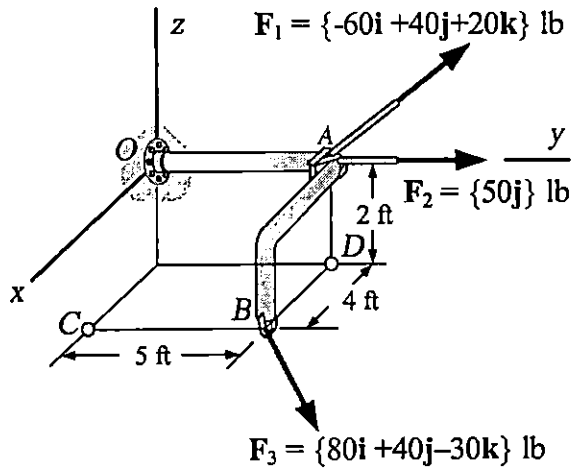
Prob. (2)

2] Determine the required length of cord *AC* in the figure so that the 8 kg lamp is suspended in the position shown. The undeformed length of the spring *AB* is $l_O = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m. [5 degrees]

3] The mast *OA* exerts a vertical force of $F = 1200$ N on the collar at *A*. Determine the tension in each of the cables for equilibrium. [5 degrees]



Prob. (3)

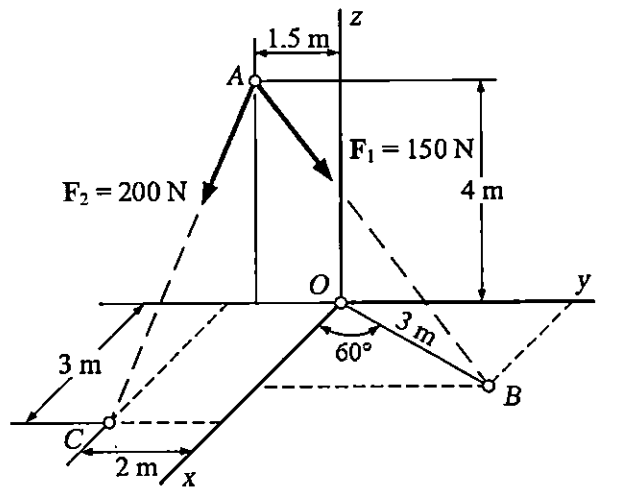


Prob. (4)

4] Three forces act on the rod shown. Determine the resultant moment they create about the flange at *O* and the coordinate direction angles of the moment axis. [5 degrees]

مع أطيب الأمنيات بالتوفيق والنجاح

1] Determine the magnitude and coordinate direction angle of the resultant force acting at point *A*.



$A(0, -1.5, 4)$, $B(1.5, 1.5\sqrt{3}, 0)$, $C(3, -2, 0)$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.098\mathbf{j} - 4\mathbf{k}}{5.92}\right)$$

$$= 38\mathbf{i} + 103.84\mathbf{j} - 101.35\mathbf{k} \text{ N}$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.025}\right)$$

$$= 119.4\mathbf{i} - 19.9\mathbf{j} - 159.2\mathbf{k} \text{ N}$$

$$\mathbf{R} = \sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = 157.4\mathbf{i} + 83.94\mathbf{j} - 260.554\mathbf{k}$$

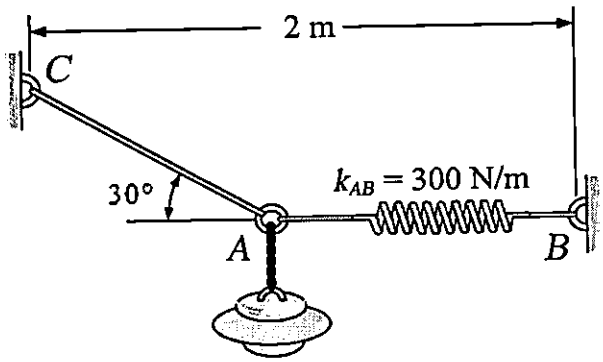
$$R = 315.77 \text{ N}$$

$$\alpha = \cos^{-1}\left(\frac{157.4}{315.77}\right) = 60.1°$$

$$\beta = \cos^{-1}\left(\frac{83.94}{315.77}\right) = 74.6°$$

$$\gamma = \cos^{-1}\left(\frac{-260.55}{315.77}\right) = 145.6°$$

2] Determine the required length of cord AC in the figure so that the 8 kg lamp is suspended in the position shown. The undeformed length of the spring AB is $l_O = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.

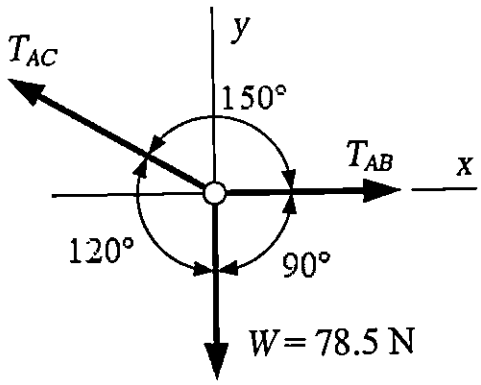


Solution I

W = 8 * 9.81 = 78.48 N

For Equilibrium of Point A

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 90°} = \frac{78.48}{\sin 150°}$$



$$T_{AB} = \frac{78.48 \sin 120°}{\sin 150°} = 135.93 \text{ N} = F_s$$

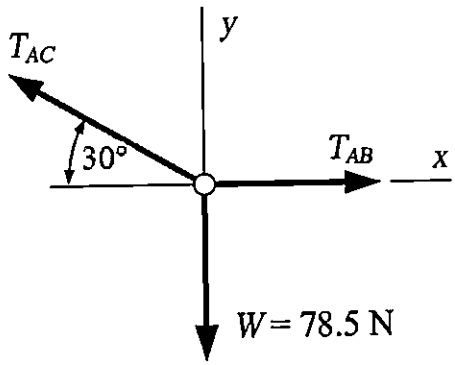
$$T_{AC} = \frac{78.48 \sin 90°}{\sin 150°} = 156.96 \text{ N}$$

$F_s = k\Delta l \qquad 135.93 = 300\Delta l$

$\Delta l = 0.453$ m

$\Delta l + 0.4 = 2 - l_{AC}\cos 30° \qquad l_{AC} = \frac{2 - 0.853}{\cos 30°} = 1.32$ m

Solution II



W = 8 * 9.81 = 78.48 N

For Equilibrium of Point A

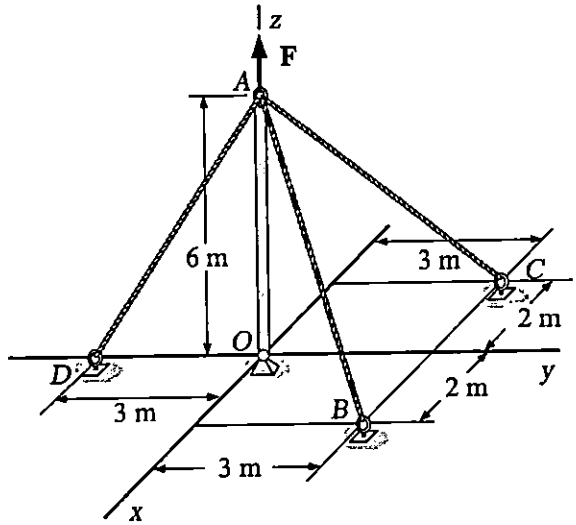
$\sum F_y = 0 \qquad T_{AC}\sin 30° - 78.48 = 0 \qquad T_{AC} = 156.96$ N

$\sum F_x = 0 \qquad T_{AB} - T_{AC}\cos 30° = 0 \qquad T_{AB} = 0.866(156.96) = 135.93 \text{ N} = F_s$

$F_s = k\Delta l \qquad 135.93 = 300\Delta l \qquad \Delta l = 0.453$ m

$\Delta l + 0.4 = 2 - l_{AC}\cos 30° \qquad l_{AC} = \frac{2 - 0.853}{\cos 30°} = 1.32$ m

3] The mast OA exerts a vertical force of $F = 1200$ N on the collar at A. Determine the tension in each of the cables for equilibrium.



$A(0, 0, 6)$, $B(2, 3, 0)$, $C(-2, 3, 0)$, $D(0, -3, 0)$

$$\mathbf{T}_B = T_B\left(\frac{2\mathbf{i}+3\mathbf{j}-6\mathbf{k}}{7}\right)$$

$$\mathbf{T}_C = T_C\left(\frac{-2\mathbf{i}+3\mathbf{j}-6\mathbf{k}}{7}\right)$$

$$\mathbf{T}_D = T_D\left(\frac{-3\mathbf{j}-6\mathbf{k}}{6.71}\right)$$

$$\mathbf{F} = 1200\mathbf{k} \text{ N}$$

$$\sum F_x = 0 \qquad \frac{2}{7}T_B - \frac{2}{7}T_C = 0 \qquad T_B = T_C \quad (1)$$

$$\sum F_y = 0 \qquad \frac{3}{7}T_B + \frac{3}{7}T_C - \frac{3}{6.71}T_D = 0 \qquad T_D = 1.9172T_B \quad (2)$$

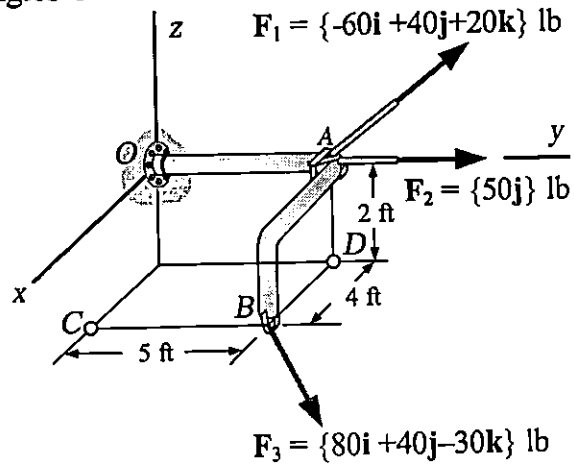
$$\sum F_z = 0 \qquad -\frac{6}{7}T_B - \frac{6}{7}T_C - \frac{6}{6.71}T_D + 1200 = 0 \quad (3)$$

From (1), (2), (3)

$$T_B = T_C = 350 \text{ N}$$

$$T_D = 671 \text{ N}$$

4] Three forces act on the rod shown. Determine the resultant moment they create about the flange at O and the coordinate direction angles of the moment axis.



$\mathbf{F}_1 = -60\mathbf{i} + 40\mathbf{j} + 20\mathbf{k}$ N $A(0, 5, 0)$

$\mathbf{F}_2 = 50\mathbf{j}$ N $A(0, 5, 0)$

$\mathbf{F}_3 = 80\mathbf{i} + 40\mathbf{j} - 30\mathbf{k}$ N $B(4, 5, -2)$

$$\mathbf{M}_{O_R} = \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_A \times \mathbf{F}_2 + \mathbf{r}_B \times \mathbf{F}_3$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ 0 & 50 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= 30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k} \text{ N.m}$$

$$M_{O_R} = \sqrt{(30)^2 + (40)^2 + (60)^2} = 78.1 \text{ N.m}$$

$$\alpha = \cos^{-1}\left(\frac{30}{78.1}\right) = 67.4°$$

$$\beta = \cos^{-1}\left(\frac{-40}{78.1}\right) = 121°$$

$$\gamma = \cos^{-1}\left(\frac{60}{78.1}\right) = 39.8°$$