Mansoura University
Faculty of Engineering

Final 1st 2019/2020
All of the New Programs
Level 000
Exam Date: 18-1- 2020

Course Name: Math 1
Course Code:MATH001
Total marks: 50 marks
Allowed Time: 2 Hours

الامتحان في صفحتين- معطي جدول للمشتقات في نهاية الصفحة الاولي – التزم الترتيب والوضوح في الحل

Solve all of the following Problems

Problem 1 (8 Marks)

Find $\frac{dy}{dx}$ of the following:

(a) $y = \cosh\left(\coth^{-1}\left(5^{(\cot^{-1}x)}\right)\right)$ (2 Marks)

(b) $y = (3^x \coth x)^{(\cosh^{-1}\sqrt{x})}$ (3 Marks)

(c) $\cosh(x+3y) = y^3 \cos x$ (3 Marks)

Problem 2 (8 Marks)

(a) At what point on the curve $y = x + e^{2x}$ is the slope of the tangent equal to three? (2 Marks)

(b) Prove that $\cosh^{-1}x = \ln\left(x \pm \sqrt{x^2-1}\right)$, for $x \geq 1$. (4 Marks)

(c) Find $\frac{dy}{dx}$ if $y = t^3 + \sin(t^2)$, $x = 3^{(\sin t)} + \sinh t$ (2 Marks)

Problem 3 (8 Marks)

(a) Find the Taylor series for $f(x) = \cos x$ about $x = \pi/6$ (4 Marks)

(b) Find $\lim_{x\to 0}\left(\frac{e^{2x} - 2x - 1}{1 - \cosh x}\right)$ (2 Marks)

(c) Find $\lim_{x\to\infty}\left(\frac{3^x}{e^x}\right)$ (2 Marks)

$f(x)$	x^n	$\sqrt{x}$	e^x	a^x	$\ln x$	$\csc x$	$\cos x$
$f'(x)$	nx^{n-1}	$1/2\sqrt{x}$	e^x	$a^x \ln a$	$1/x$	$-\csc x \cot x$	$-\sin x$

$f(x)$	$\cosh x$	$\sinh x$	$\coth x$	$\coth^{-1}x$	$\cot^{-1}x$	$\cosh^{-1}x$	$\cos^{-1}x$
$f'(x)$	$\sinh x$	$\cosh x$	$-\text{csch}^2 x$	$\frac{-1}{x^2-1}$	$\frac{-1}{1+x^2}$	$\frac{1}{\sqrt{x^2-1}}$	$\frac{-1}{\sqrt{1-x^2}}$

Problem 4 (8 Marks)

Express $\dfrac{4x^3 + 23\,x^2 + 45\,x + 27}{x^3 + 5\,x^2 + 8\,x + 4}$ as the sum of its partial fractions

Problem 5 (8 Marks)

For the function $f(x) = x^4 - x^3 - x^2 + ax + b$

If $x = 1$ is a root of multiplicity $(m > 1)$

i) Find a, b and m. (using the synthetic division).

ii) Find the other roots.

iii) Sketch $f(x)$

Problem 6 (10 Marks)

(a) Using Gauss-Jordan elimination method to find A^{-1} (5 Marks)

$$\text{if} \quad A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{pmatrix}$$

And then solve the system

$$\begin{aligned} y + z &= 1 \\ x + 2z &= 0 \\ -x + y &= 1 \end{aligned}$$

(b) Find the value(s) of the constant k such that the following system

$$\begin{aligned} x_1 + 4x_2 + 5x_3 &= 4 \\ x_1 + k\,x_2 + 7x_3 &= 6 \\ x_1 + 4x_2 + (k+3)\,x_3 &= 6 \end{aligned}$$

(5 Marks)

has i) unique solution ii) no solution iii) many solutions.

انتهت الأسئلة مع الدعوات لكم بالتوفيق

Model Answer

Problem 1 (8 Marks)

Find $\frac{dy}{dx}$ of the following:

(a) $y = \cosh\left(\coth^{-1}\left(5^{(\cot^{-1}x)}\right)\right)$ (2 Marks)

Answer: $\frac{dy}{dx} = \sinh\left((\coth^{-1}\left(5^{(\cot^{-1}x)}\right)\right) \frac{5^{(\cot^{-1}x)} \ln 5}{5^{2(\cot^{-1}x)}-1} \frac{1}{1+x^2}$

(b) $y = (3^x \coth x)^{(\cosh^{-1}\sqrt{x})}$ (3 Marks)

Answer: $\frac{dy}{dx} = (3^x \coth x)^{(\cosh^{-1}\sqrt{x})} \left(\cosh^{-1}\sqrt{x} \left(\ln 3 - \frac{\operatorname{csch}^2 x}{\coth x}\right) + (x \ln 3 + \ln \coth x) \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x-1}}\right)$

(c) $\cosh(x+3y) = y^3 \cos x$ (3 Marks)

Answer: $\sinh(x+3y)\left(1+\frac{3dy}{dx}\right) = 3y^2 \frac{dy}{dx}\cos x - y^3 \sin x$

$$\frac{dy}{dx} = \frac{-y^3 \sin x - \sinh(x+3y)}{3\sinh(x+3y) - 3y^2 \cos x}$$

Problem 2 (8 Marks)

(a) At what <u>point</u> on the curve $y = x + e^{2x}$ is the slope of the tangent equal to three? (2 Marks)

Answer: $y' = 1 + 2e^{2x} \rightarrow 3 = 1 + 2e^{2x} \rightarrow x = 0 \rightarrow y = 1 \rightarrow$ (0,1)

(b) Prove that $\cosh^{-1} x = \ln\left(x \pm \sqrt{x^2-1}\right)$, for $x \geq 1$. (4 Marks)

Answer: $x = \cosh y = \frac{e^y - e^{-y}}{2} \rightarrow e^{2y} - 2xe^y + 1 = 0$

$$e^y = \frac{2x \pm \sqrt{4x^2-4}}{2} \rightarrow e^y = x \pm \sqrt{x^2-1} \xrightarrow{ln}$$

$$y = \ln\left(x \pm \sqrt{x^2-1}\right), \quad x \geq 1$$

(c) Find $\frac{dy}{dx}$ if $y = t^3 + \sin(t^2)$, $x = 3^{(\sin t)} + \sinh t$ (2 Marks)

Answer: $\frac{dy}{dt} = 3t^2 + 2t\cos t^2$, $\frac{dx}{dt} = 3^{(\sin t)} \ln 3 \cos t + \cosh t$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{3t^2 + 2t\cos t^2}{3^{(\sin t)} \ln 3 \cos t + \cosh t}$$

Problem 3 (8 Marks)

(a) Find the Taylor series for $f(x) = \cos x$ about $x = \pi/6$ (4 Marks)

Answer: $f(x) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2 + \cdots$

(b) Find $\lim_{x\to 0}\left(\frac{e^{2x} - 2x - 1}{1 - \cosh x}\right)$ (2 Marks)

Answer: using L'Hopital, $\lim_{x\to 0}\left(\frac{e^{2x} - 2x - 1}{1 - \cosh x}\right) = -4$

(c) Find $\lim_{x\to\infty}\left(\frac{3^x}{e^x}\right)$ (2 Marks)

Answer: since $\frac{3}{e} > 1$, $\lim_{x\to\infty}\left(\frac{3}{e}\right)^x = \infty$

$f(x)$	x^n	$\sqrt{x}$	e^x	a^x	$\ln x$	$\csc x$	$\cos x$
$f'(x)$	nx^{n-1}	$1/2\sqrt{x}$	e^x	$a^x \ln a$	$1/x$	$-\csc x \cot x$	$-\sin x$

$f(x)$	$\cosh x$	$\sinh x$	$\coth x$	$\coth^{-1} x$	$\cot^{-1} x$	$\cosh^{-1} x$	$\cos^{-1} x$
$f'(x)$	$\sinh x$	$\cosh x$	$-\text{csch}^2 x$	$\frac{-1}{x^2 - 1}$	$\frac{-1}{1 + x^2}$	$\frac{1}{\sqrt{x^2 - 1}}$	$\frac{-1}{\sqrt{1 - x^2}}$

Problem 4 (8 Marks)

Exprees $\frac{4x^3 + 23x^2 + 45x + 27}{x^3 + 5x^2 + 8x + 4}$ as the sum of its partial fractions

Answer:

$$\frac{4x^3 + 23x^2 + 45x + 27}{x^3 + 5x^2 + 8x + 4} = 4 + \frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{(x+2)^2}$$

Problem 5 (8 Marks)

For the function $f(x) = x^4 - x^3 - x^2 + ax + b$

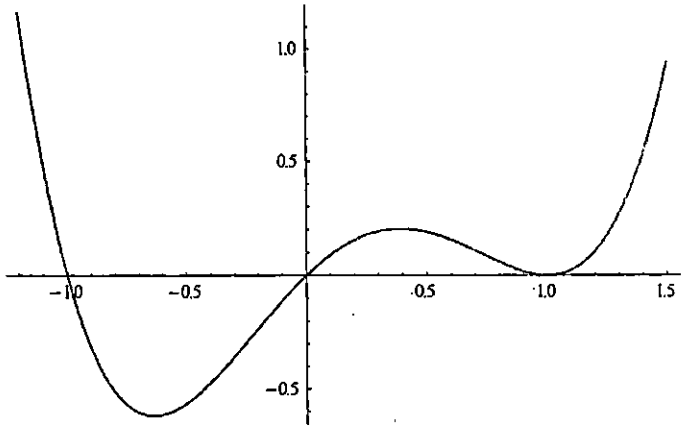
If $x = 1$ is a root of multiplicity $(m > 1)$

i) Find a, b and m. (using the synthetic division).

ii) Find the other roots.

iii) Sketch $f(x)$

Answer: i) $a = 1,\ b = 0, m = 2$ ii) $x = 0, -1$ iii)



Problem 6 (10 Marks)

(a) Using Gauss-Jordan elimination method to find A^{-1} (5'Marks)

$$\text{if} \quad A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{pmatrix}$$

And then solve the system

$$\begin{aligned} y + z &= 1 \\ x + 2z &= 0 \\ -x + y &= 1 \end{aligned}$$

Answer:

$$A^{-1} = \begin{pmatrix} 2 & -1 & -2 \\ 2 & -1 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

$$x = 0, \quad y = 1, \quad z = 0$$

(b) Find the value(s) of the constant k such that the following system

$$\begin{aligned} x_1 + 4x_2 + 5x_3 &= 4 \\ x_1 + k\,x_2 + 7x_3 &= 6 \\ x_1 + 4x_2 + (k+3)\,x_3 &= 6 \end{aligned}$$

(5 Marks)

has i) unique solution ii) no solution iii) many solutions.

Answer:

i) unique solution $k \neq 2,4$

ii) no solution $k = 2$

iii) many solutions $k = 4$.