

Name (in Arabic):

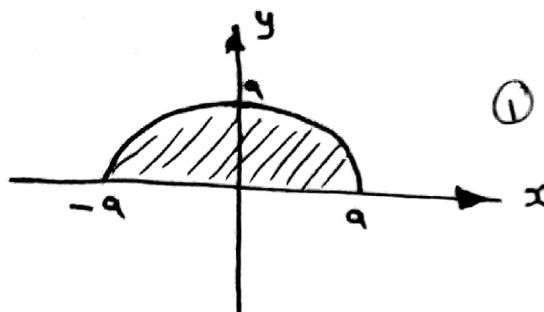
1. [3 marks] Prove that area of the circle $x^2 + y^2 = a^2$ is πa^2 .

$$y^2 = a^2 - x^2$$

$$y = \pm \sqrt{a^2 - x^2}$$

eqn. of upper half of circle

$$\text{is } y = \sqrt{a^2 - x^2}$$



Area of half circle

$$= \int_{-a}^a \sqrt{a^2 - x^2} dx \quad (1)$$

$$= 2 \int_0^a \sqrt{a^2 - x^2} dx \quad \text{even fn.}$$

$$= 2 \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2 \theta} \cdot a \cos \theta d\theta$$

$$= 2a^2 \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta$$

$$= \frac{2a^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta$$

$$= a^2 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= a^2 \left[\left(\frac{\pi}{2} + \frac{\sin(\pi)}{2} \right) - \left(0 + \frac{\sin 0}{2} \right) \right]$$

$$= \boxed{\frac{\pi a^2}{2}} \quad (1)$$

$$\text{area of circle} = 2 \left(\frac{\pi a^2}{2} \right)$$

$$= \pi a^2 \quad \#$$

2. [2 marks] Evaluate $\int_0^2 \sqrt{2x - x^2} dx$

$$I = \int_0^2 \sqrt{-(x^2 - 2x)} dx$$

$$= \int_0^2 \sqrt{-[(x-1)^2 - 1^2]} dx$$

$$= \int_0^2 \sqrt{1 - (x-1)^2} dx$$

$$= \int_{-1}^1 \sqrt{1 - u^2} du \quad (1)$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 \theta} \cdot C_s \theta d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} C_s^2 \theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} C_s^2 \theta d\theta \quad \text{even fn.}$$

$$= 2 \left(\frac{1}{2} \right) \int_0^{\frac{\pi}{2}} (1 + C_s 2\theta) d\theta$$

$$= \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{2} + \frac{\sin(\pi)}{2} \right) - \left(0 + \frac{\sin 0}{2} \right)$$

$$= \boxed{\frac{\pi}{2}} \quad (1)$$

$$\begin{aligned} u &= x-1 \\ du &= dx \\ x=0 &\Rightarrow u=-1 \\ x=2 &\Rightarrow u=1 \end{aligned}$$

$$\begin{aligned} u &= \sin \theta \\ du &= C_s \theta d\theta \\ \sqrt{ } &= C_s \theta \\ u=-1 &\Rightarrow \theta=\frac{\pi}{2} \\ u=1 &\Rightarrow \theta=\frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} x &= a \sin \theta \\ dx &= a C_s \theta d\theta \\ x=0 &\Rightarrow \theta=0 \\ x=a &\Rightarrow \theta=\frac{\pi}{2} \\ \sqrt{ } &= a C_s \theta \end{aligned}$$

[2 marks] Discuss the convergence or divergence of the improper integral $\int_{-\infty}^{-1} \frac{1}{x} dx$.

$$\begin{aligned}
 I &= \lim_{c \rightarrow -\infty} \int_c^{-1} \frac{1}{x} dx \quad (1) \\
 &= \lim_{c \rightarrow -\infty} [\ln|x|]_c^{-1} \\
 &= \lim_{c \rightarrow -\infty} [\ln|-1| - \ln|c|] \\
 &= \lim_{c \rightarrow -\infty} [\ln 1 - \ln|c|] \\
 &= -\ln|1-\infty| \\
 &= -\ln \infty \\
 &= -\infty
 \end{aligned}$$

integral is divergent (1)

[2 marks] Find $\int \cosh(x) \ln(\sinh x) dx$

$$\begin{aligned}
 u &= \sinh x \\
 du &= \cosh x dx \\
 I &= \int \ln(u) du \quad (1) \\
 \frac{d}{du} &\quad \int du \\
 \ln u &\quad + \\
 \frac{1}{u} &\quad \cancel{-f} \rightarrow u
 \end{aligned}$$

$$\begin{aligned}
 I &= u \ln u - \int \frac{1}{u} \cdot u du \\
 &= u \ln u - \int 1 du \\
 &= u \ln u - u + C
 \end{aligned}$$

$$I = \sinh x \ln(\sinh x) - \sinh x + C \quad (1)$$

[2 marks] If $f(x)$ is even function, $g(x)$ is odd function, $\int_{-1}^0 f(x) dx = 3$ and $\int_4^1 f(x) dx = 2$. Evaluate (i) $\int_{-1}^1 f(x)g(x) dx = \boxed{0}$ (2) $f(x) \cdot g(x)$ is odd fn.

$$\begin{aligned}
 \text{(ii)} \int_{-1}^1 [2f(x) + 3g(x)] dx &= 2 \int_{-1}^1 f(x) dx + 3 \int_{-1}^1 g(x) dx \\
 &= 4 \int_0^1 f(x) dx = 4 \int_{-1}^0 f(x) dx = \boxed{12} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \int_{-4}^4 f(x) dx &= 2 \int_0^4 f(x) dx \\
 &= 2 \left[\int_0^1 f(x) dx + \int_1^4 f(x) dx \right] \\
 &= 2 \left[\int_{-1}^0 f(x) dx - \int_{-4}^1 f(x) dx \right] \\
 &= 2 [3 - 2] = \boxed{2} \quad (1)
 \end{aligned}$$

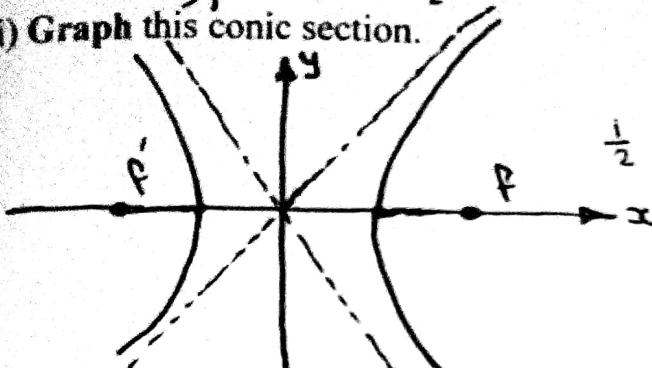
[3 marks] For the conic section

$$4x^2 - y^2 = 16$$

(i) Write the name of this conic section.

hyperbola $\frac{1}{2}$

(ii) Graph this conic section.



(iii) Describe this conic section.

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

\times hyperbola $a^2 = 4$ $a = 2$

$$e = \sqrt{1 + \frac{16}{4}} = \sqrt{5} \quad b^2 = 16 \quad b = 4$$

$$ae = 2\sqrt{5}$$

$$\frac{a}{e} = \frac{2}{\sqrt{5}}$$

Center	$(0, 0)$
length of transverse axis	4
length of conjugate axis	8
e	$\sqrt{5}$
ν	$(\pm 2, 0)$
F	$(\pm 2\sqrt{5}, 0)$
DD'	$x = \pm \frac{2}{\sqrt{5}}$
eqn. of asymptotes	$\frac{x^2}{4} - \frac{y^2}{16} = 0$ $y = \pm 2x$
eqn. of focal axis	$y = 0$

7. [3 marks] Write the standard form of the ellipse with eccentricity $e = \frac{4}{5}$ and the ends of the major axis are the center of the circle $x^2 + y^2 - 10x + 21 = 0$ and the vertex of parabola $y^2 + 16x + 80 = 0$.

Circle

$$C(5, 0) \quad \textcircled{1}$$

parabola

$$y^2 = -16x - 80$$

$$y^2 = -16(x+5)$$

axes translation at $(-5, 0)$

$$Y = y \quad X = x + 5$$

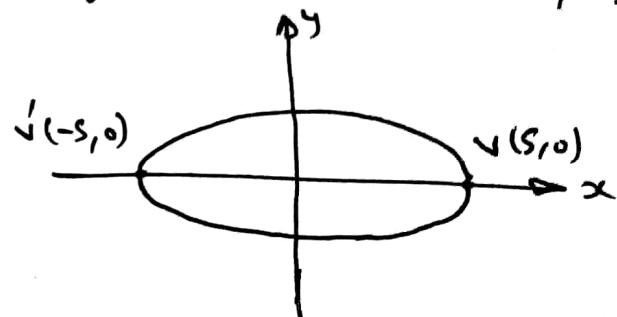
$$\text{new eqn. } Y^2 = -16X$$

$\nu(0, 0)$ new axes

$\nu(-5, 0)$ old axes $\textcircled{1}$

ellipse has $e = \frac{4}{5}$

ends of major axis are $(5, 0), (-5, 0)$



$C(0, 0)$

\times -ellipse

$$2a = 10 \rightarrow a = 5 \quad \textcircled{2}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\frac{16}{25} = 1 - \frac{b^2}{25}$$

$$16 = 25 - b^2$$

$$b^2 = 9 \quad \textcircled{3}$$

eqn. of ellipse

$$\frac{(x-0)^2}{25} + \frac{(y-0)^2}{9} = 1$$

$\textcircled{4}$

[2 marks] Given the parabola

$$y^2 - 2y - 4x + 9 = 0.$$

Find the equation of the tangent line at the point whose coordinate $y = -1$.

$$y = -1 \rightarrow 1 + 2 - 4x + 9 = 0$$

$$4x = 12 \quad \boxed{x = 3}$$

point of tangency is

$$(x_1, y_1) = (3, -1)$$

$$x^2 \rightarrow 3x$$

$$y^2 \rightarrow -y$$

$$x \rightarrow \frac{x+3}{2} \quad \textcircled{1}$$

$$y \rightarrow \frac{y-1}{2}$$

eqn. of tangent line at $(3, -1)$

$$-y - 2\left(\frac{y-1}{2}\right) - 4\left(\frac{x+3}{2}\right) + 9 = 0$$

$$-y - y + 1 - 2x - 6 + 9 = 0$$

$$-2y - 2x + 4 = 0 \quad \div -2$$

$$\boxed{x + y - 2 = 0}$$

$\textcircled{\frac{1}{2}}$

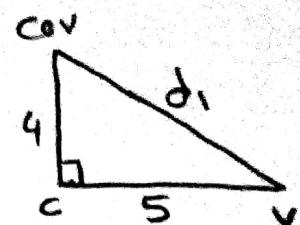
[3 marks] Given the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

(i) Find the distance between vertex and co-vertex.

$$d_1 = \sqrt{25 + 16}$$

$$= \boxed{\sqrt{41}} \quad \textcircled{1}$$



$$a^2 = 25 \quad a = 5$$

$$b^2 = 16 \quad b = 4$$

(ii) Find the area of triangle whose vertices are the two foci and co-vertex.

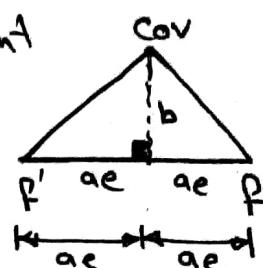
$$e = \sqrt{1 - \frac{16}{25}} = \boxed{\frac{3}{5}}$$

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} (2ae) \cdot b$$

$$= 5 \left(\frac{3}{5}\right) (4)$$

$$= \boxed{12} \quad \textcircled{1}$$



(iii) Find the distance between focus and directrix.

$$d_2 = \frac{a}{e} - ae$$

$$= \frac{5}{(\frac{3}{5})} - 5 \left(\frac{3}{5}\right)$$

$$= \frac{25}{3} - 3$$

$$= \boxed{\frac{16}{3}} \quad \textcircled{1}$$

Draft

