



Final Exam

Digital Design

Time Allowed: 2 hrs.
Dr: Ahmed Saleh
Total Marks: 50
MTE, BME Students.
2017 - 2018



Solve the following:

Question 1

- يسمح باستخدام القلم الرصاص (شرط وضوح الخط).
- الرجاء وضوح الرسم قدر المستطاع (ليس شرطاً استخدام المسطرة)
- الامتحان في ورقتين.
- عدد الاسئلة = 3

(A) **Using Equations**, simplify the following function:

- $F(X,Y,Z) = XY + X'Z + YZ$
- $F(X,Y,Z) = (X+Y)(X'+Z)(Y+Z)$

(B) **Using a Map**, simplify the following function:

$$F(A,B,C,D) = BCD + A'D' + A'B'D$$

Use:

- Sum of product form.
- Product of sum form.

(C) **Implement (Draw) the function** $F(A,B,C,D) = (A'BD + CD + A'B'C)BD'$ Using:

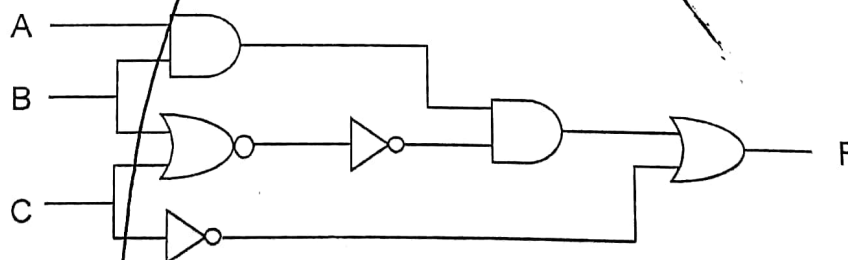
- AND – NOT only.
- OR – NOT only.

(D) **Find the complement of the following function:** $F(A,B,C,D) = B'D + A'BC' + A'BC + A'D$

(E) **Using Equations:**

- Express the function: $F(A,B,C) = A + B'C$ in **Sum of minterms**, what are the corresponding Maxterms?
- Express the function: $F(A,B,C) = (A+B)(B'+C)$ in **Product of Maxterms**, what are the corresponding minterms?

(F) **Find the Truth table for the following circuit:**



(G) **Assuming** four variables in the order (A, B, C, D), Simplify the following functions (F1, F2, F3, F4) using the shown maps

1	0	x	1
x	x	1	1
x	1	0	1
1	0	0	x

F1

1	1	1	1
x	1	1	x
1	x	0	1
x	0	0	x

F2

1	0	x	0
x	x	0	1
x	x	x	1
1	0	1	0

F3

1	x	x	1
x	x	1	1
1	x	x	1
1	x	x	x

F4

(27 marks)



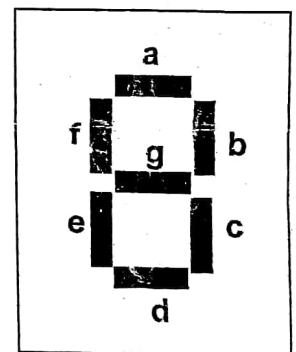
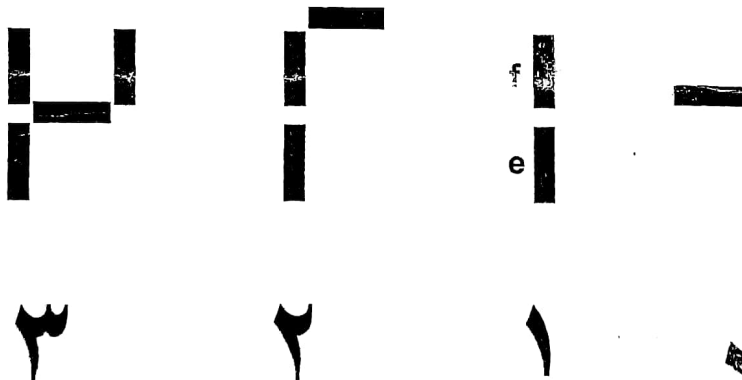
Question 2

- (A) Using Decoder and OR gates, Design digital circuit that accepts 4 bit input number A (i.e., $A=A_3A_2A_1A_0$). The circuit has 3 outputs: X , Y , and Z . so that:
- $X=1$ if the number of ones in the input number Greater than the number of zeros.
 - $Y=1$ if the number of ones in the input number Less than the number of zeros.
 - $Z=1$ if the number of ones in the input number Equals the number of zeros.
- (B) Define:
- 3-bit parallel Adder.
 - 2-bit comparator.
- (C) Show the internal structure of 2X1 Multiplexer.
- (D) Implement the function $F(A,B,C)=A'B+BC$. Using: **Decoder** and an **And** gate,

Question 3

(17 marks)

Design a digital circuit that accepts 2-bit binary number and the output is the numbers (0, 1, 2, 3) appeared on the seven segments in the Arabic fashion as shown in the following figure. (Note: the only used of the seven segments are the segments: a, b, e, f, g).



(6 marks)

----- End of Questions -----

With Best Wishes
Dr. Ahmed Saleh
Plz, send feedback about the exam to:
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Final Exam Digital Design - Solution -

Question (1)

(A) (i)
$$\begin{aligned} F &= xy + \bar{x}z + yz \\ &= xy + \bar{x}z + yz(x + \bar{x}) \\ &= xy + xyx + \bar{x}z + \bar{x}yz \\ &= xy(1 + z) + \bar{x}z(1 + y) \\ &= xy + \bar{x}z \end{aligned}$$

(ii)
$$\begin{aligned} F &= ((x+y)(\bar{x}+z)(y+z))' \\ &= ((x+y)' + (\bar{x}+z)' + (y+z)')' \\ &= (x'y' + x\bar{z} + \bar{y}z)' \\ &= (\bar{x}\bar{y} + x\bar{z} + \bar{y}z(x+\bar{x}))' \\ &= (\bar{x}\bar{y}(1+\bar{z}) + x\bar{z}(1+\bar{y}))' \\ &= (\bar{x}\bar{y} + x\bar{z})' \\ &= (x+y)(\bar{x}+z) \end{aligned}$$

□

$$(B) \quad F = BCD + \bar{A}\bar{D}' + \bar{A}'B'D$$

C \ D		0		1	
		0	1	1	0
A \ B	0	1	1	1	1
	1	1	0	1	1
1	1	0	0	1	0
	0	0	0	0	0

$$\begin{aligned} \text{Sum of product} &= \bar{A}\bar{B} + \bar{A}\bar{D} + BCD \\ &= \sum (0, 1, 2, 3, 4, 6, 7, 15) \end{aligned}$$

$$\begin{aligned} \text{Product of sum} &= (A+B)(A+D)(B+\bar{C}+D) \\ &= \prod (5, 8, 9, 10, 11, 12, 13, 14) \end{aligned}$$

$$\begin{aligned} (C) \quad F(A, B, C, D) &= (\bar{A}BD + CD + \bar{A}B'C)BD \\ &= \bar{A}BDBD' + CDBD' + \bar{A}B'cBD' \\ &= 0 + 0 + 0 \\ &= 0 \end{aligned}$$

2

(D)

$$\begin{aligned} F &= (B'D + A'BC' + A'BC + A'D)' \\ &= (B'D)' \cdot (A'BC')' \cdot (A'BC)' \cdot (A'D)' \\ &= (B + D)(A + B + C)(A + B' + C)(A + D') \end{aligned}$$



(E)(i) $F = A + B'C$

$$= A(B + B')(C + C') + (A + A')(BC)$$

$$= A(BC + B'C + BC' + B'C') + ABC + A'BC$$

$$= ABC + AB'C + ABC' + AB'C' + \cancel{ABC} + A'BC$$

$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}$$

$$F = \sum (m_1, m_4, m_5, m_6, m_7)$$

$$\prod (M_0, M_2, M_3)$$

(ii) $F(A, B, C) = (A + B)(B' + C)$

$$= ((A + B) + CC')((B' + C) + AA')$$

$$= \cancel{ABC} + A$$

$$= (A + B + C)(A + B + C')(A + B' + C)(A + B' + C')$$

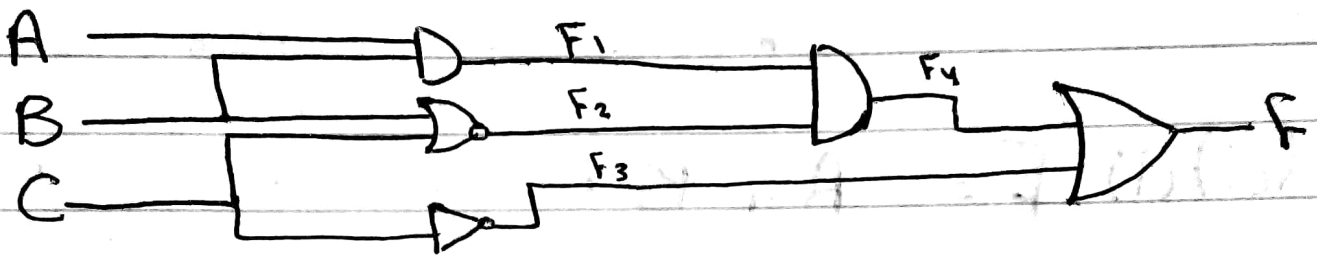
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$$F = \prod (M_0, M_1, M_2, M_6)$$

$$= \sum (m_3, m_4, m_5, m_7)$$

(F)

Find Truth table



A	B	C	F ₁	F ₂	F ₃	F ₄	F
0	0	0	0	1	1	0	1
0	0	1	0	0	0	0	0
0	1	0	0	0	1	0	1
0	1	1	0	0	0	0	0
1	0	0	0	1	1	0	1
1	0	1	0	0	0	0	0
1	1	0	0	0	1	0	1
1	1	1	1	0	0	0	0

(G)

AB \ CD	00	01	11	10
00	1	0	x	1
01	x	x	1	1
11	x	1	0	1
10	1	0	0	x

AB \ CD	00	01	11	10
00	1	1	1	1
01	x	1	1	x
11	1	x	0	1
10	x	0	0	x

$$F_1 = \bar{D} + \bar{C}B + \bar{A}B$$

$$F_2 = \bar{A} + \bar{D}$$

AB \ CD	00	01	11	10
00	1	0	x	0
01	x	x	0	1
11	x	x	x	1
10	1	0	1	0

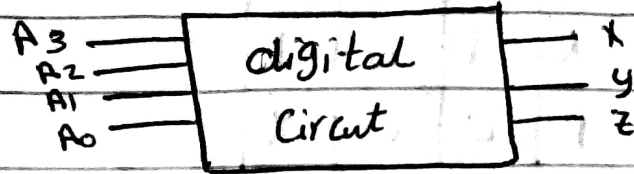
AB \ CD	00	01	11	10
00	1	x	x	1
01	x	x	1	1
11	1	x	x	1
10	1	x	x	x

$$F_3 = \bar{C}\bar{D} + B + ACD$$

$$F_4 = 1$$

Question 2

(A)

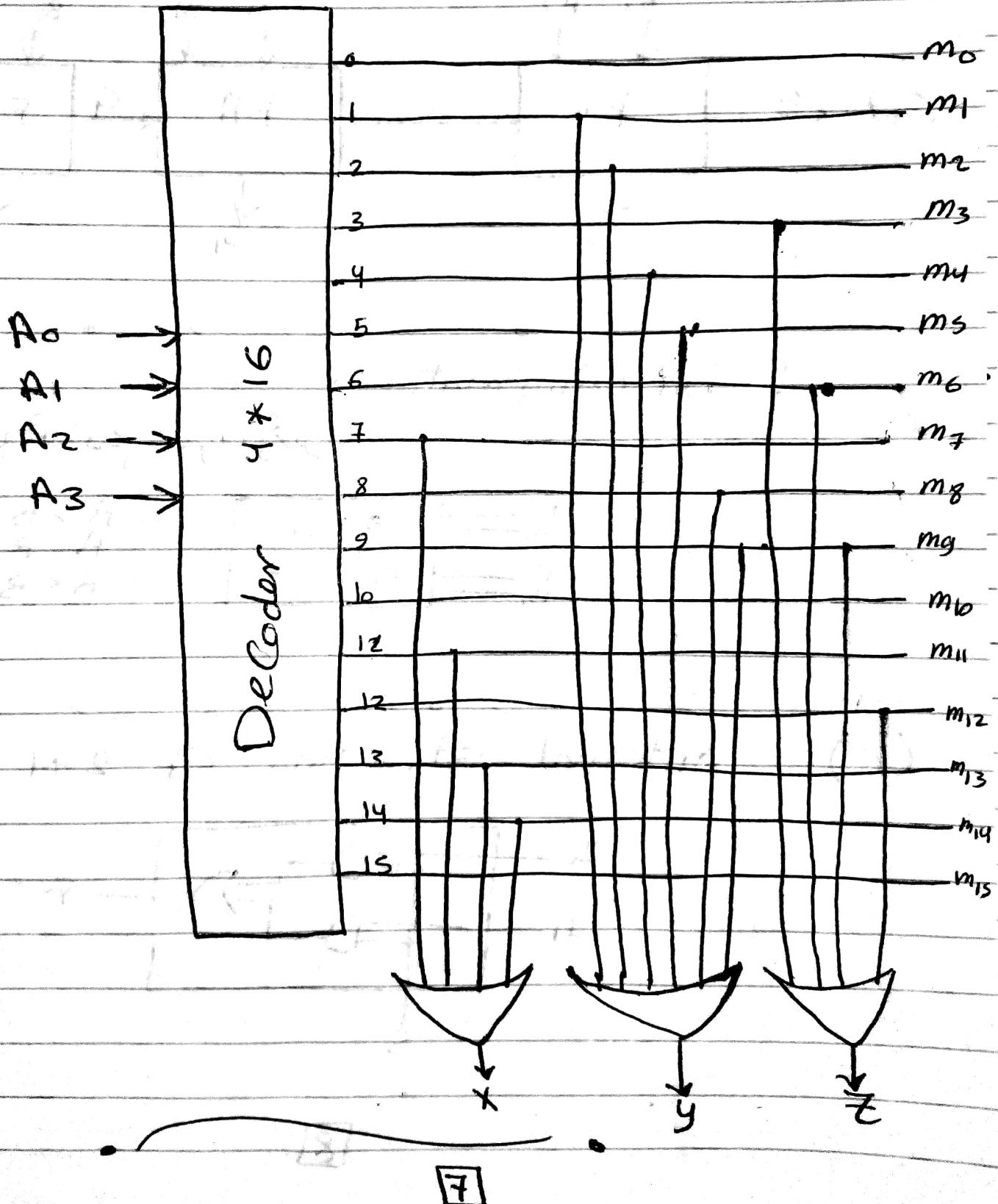


A_0	A_1	A_2	A_3	x	y	z	minterms
0	0	0	0	0	0	0	m_0
0	0	0	1	0	1	0	m_1
0	0	1	0	0	1	0	m_2
0	0	1	1	0	0	1	m_3
0	1	0	0	0	1	0	m_4
0	1	0	1	0	1	0	m_5
0	1	1	0	0	0	1	m_6
0	1	1	1	1	0	0	m_7
1	0	0	0	0	1	0	m_8
1	0	0	1	0	1	1	m_9
1	0	1	0	0	0	0	m_{10}
1	0	1	1	1	0	0	m_{11}
1	1	0	0	0	0	1	m_{12}
1	1	0	1	1	0	0	m_{13}
1	1	1	0	1	0	0	m_{14}
1	1	1	1	0	0	0	m_{15}

$$X = m_7 + m_{11} + m_{13} + m_{14} = \Sigma(7, 11, 13, 14)$$

$$Y = m_1 + m_2 + m_4 + m_5 + m_8 + m_9 = \Sigma(1, 2, 4, 5, 8, 9)$$

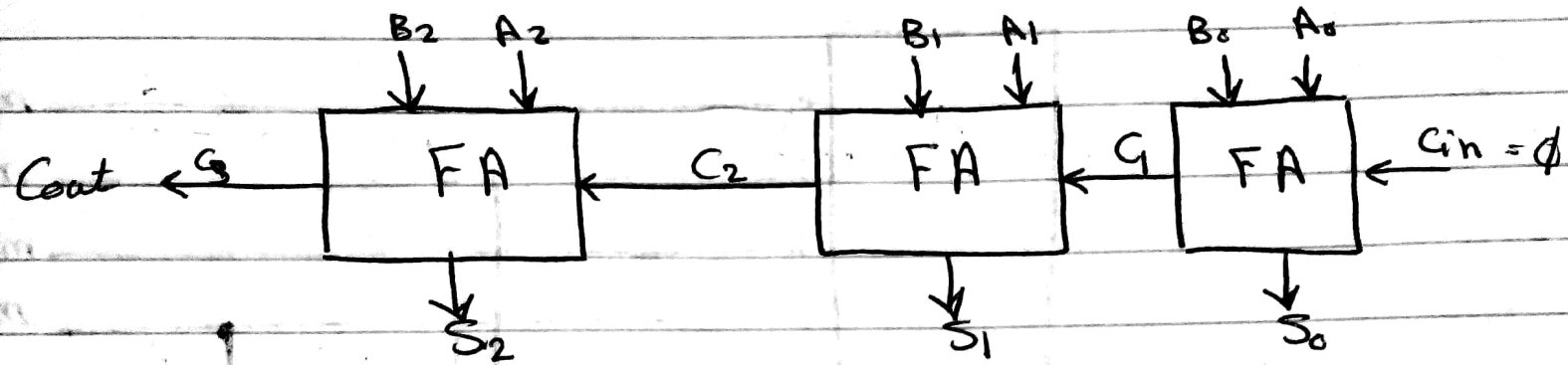
$$Z = m_3 + m_6 + m_9 + m_{12} = \Sigma(3, 6, 9, 12)$$



(B)

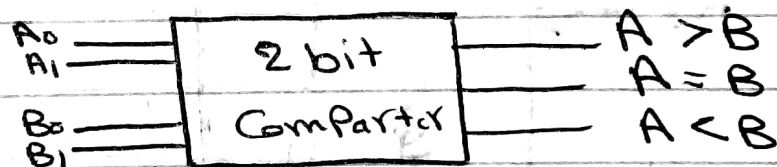
3-bit adder:

هو جهاز يقوم بجمع رقمين كل رقم مكون من 3 bit وبالتالي فان الناتج قد يصل الى $4 \text{ bit} = 3 \text{ bit} + 1$

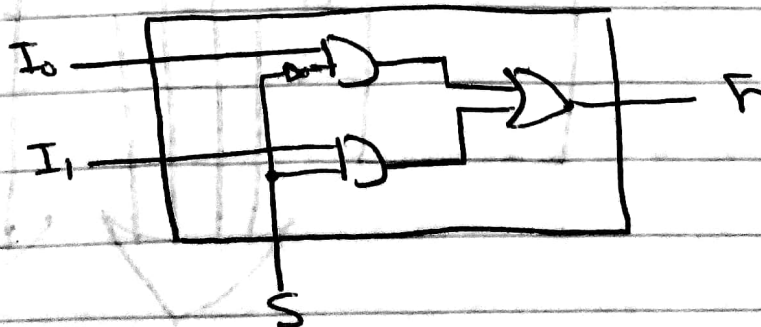


2-bit Comparator:

دائرة رقمية تستخدم لمقارنة رقمين



(c) internal structure of 2×1 mux



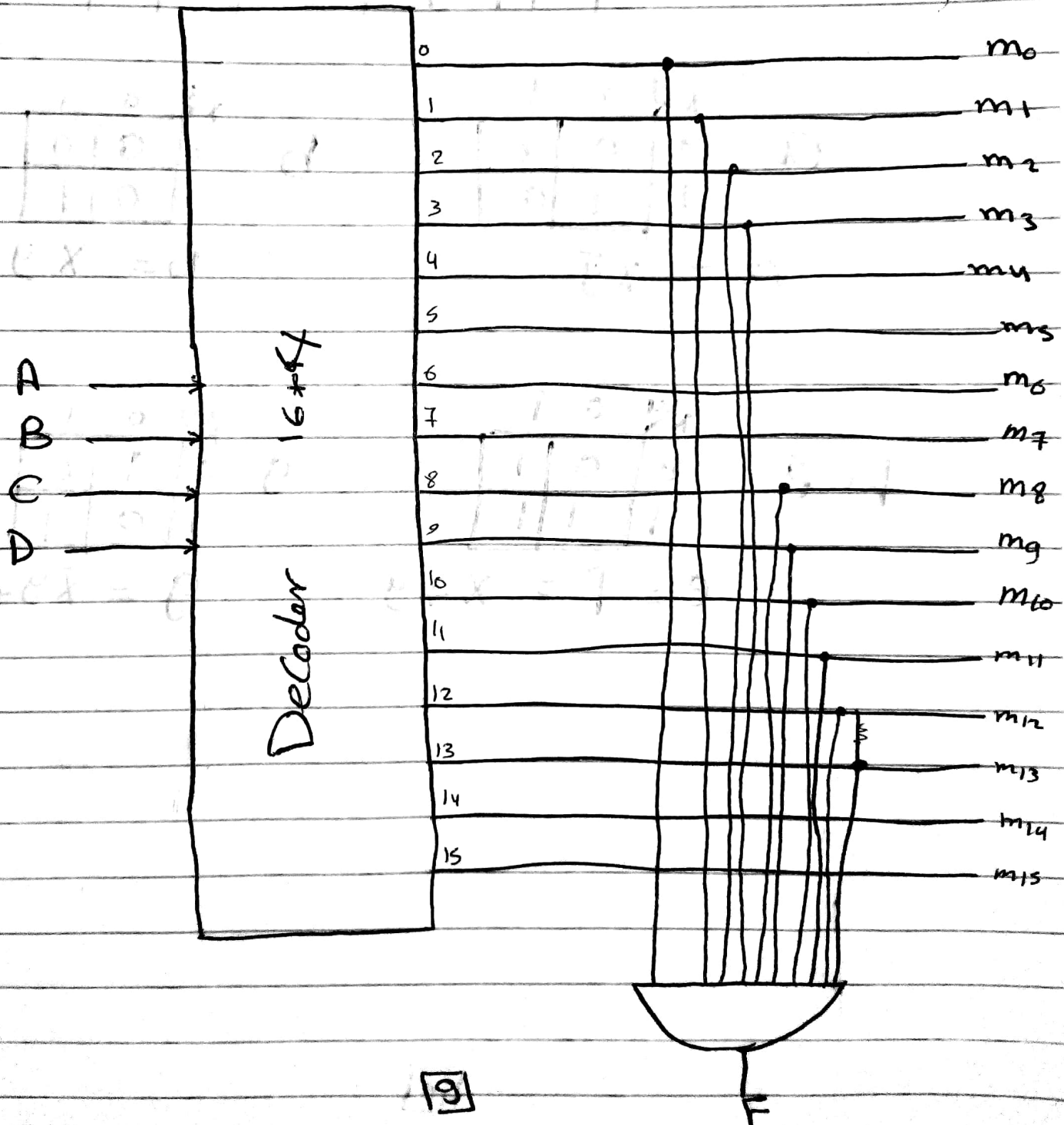
(D)

$$F(A, B, C, D) = A'B + BC$$

$$F = \sum (4, 5, 6, 7, 14, 15)$$

$$= \prod (0, 1, 2, 3, 8, 9, 10, 11, 12, 13)$$

A \ B \ C \ D	0 0	0 1	1 0	1 1
0 0	0	0	0	0
0 1	1	1	1	1
1 0	0	0	1	1
1 1	0	0	0	0



Q. 3)

	x	y	a	b	e	f	g
0	0	0	0	0	0	0	1
1	0	1	0	0	1	1	0
2	1	0	1	0	1	1	0
3	1	1	0	1	1	1	1

a

xy	0	1
0	0	0
1	1	0

$$a = x\bar{y}$$

b

xy	0	1
0	0	0
1	0	1

$$b = xy$$

f & e

xy	0	1
0	0	1
1	1	1

$$e = f = x + y$$

g

xy	0	1
0	1	0
1	0	1

$$g = \bar{x}y + x\bar{y}$$

