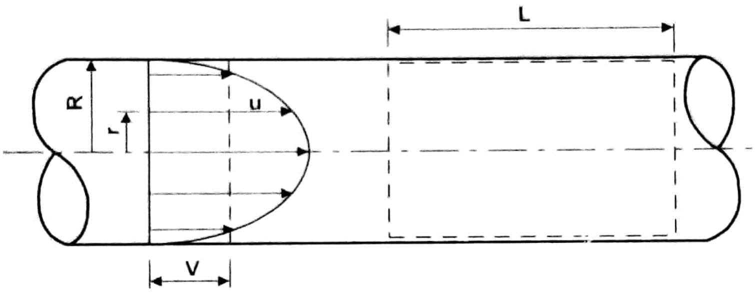
Mansoura University Faculty of Engineering Program of Biomedical Engineering. Course Title: Fluid Mechanics Course Code: MPE 171		Level: 100 Exam Type: Final Date: 3 January 2016 Time: 2 Hours Full Mark: 50

Answer all the following questions.

Question (1) [8 Marks]

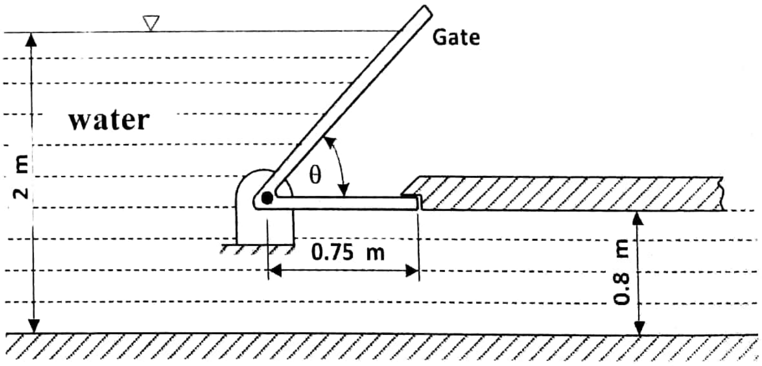
The velocity distribution of laminar flow in a circular pipe is given by $u = 2V\left(1 - \frac{r^2}{R^2}\right)$,

where *R* is the pipe radius, and *V* is the average flow velocity. Find the shear stress at the pipe wall, the wall shear force and the friction power loss along a length of pipe *L*.



Question (2) [14 Marks]

(a) A hinged gate is used as a retainer for water as shown in Fig. The liquid depth to the horizontal portion of the gate is 0.8 m, and the gate itself is to be designed so that the oil depth does not exceed 2 m. When the depth is greater than 2 m, the fluid forces act to open the gate, and some oil escapes through it. The gate width is 1 m . Determine the angle θ required for the gate to open when necessary.



(b) Consider a wooden cylinder (sg=0.6) 1 m in diameter and 0.8 m long. Would this cylinder be stable if placed to float with its axis vertical in oil (sg=0.8)?

Question (3) [8 Marks]

(a) Drive the following formula to determine the time to lower the liquid level in a tank of a constant cross section by means of an orifice.

$$t = \frac{2A}{cd\ a\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$

Where: A is the area of tank, a is the area of small orifice, cd is the discharge coefficient , H_1 is the liquid level at time=0 and H_2 is the liquid level at time t.

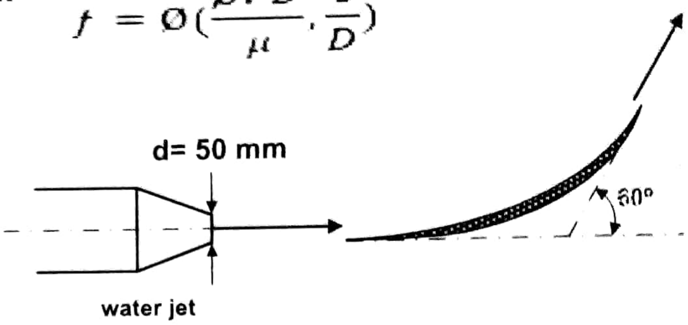
(b) Leakage in the bottom of a storage tank for oil is found to cause the surface level to decrease from 3 m to 2.7m in 24 hours. What further drop in level will occur during an additional 5 days?

Question (4) [10 Marks]

(a) The friction factor f for fluid flow in a rough pipe is a function of average flow velocity V, density ρ, viscosity μ, roughness ε and diameter D: $f = \emptyset(V, \rho, \mu, \varepsilon, D)$
Use the pi theorem to drive the following form.

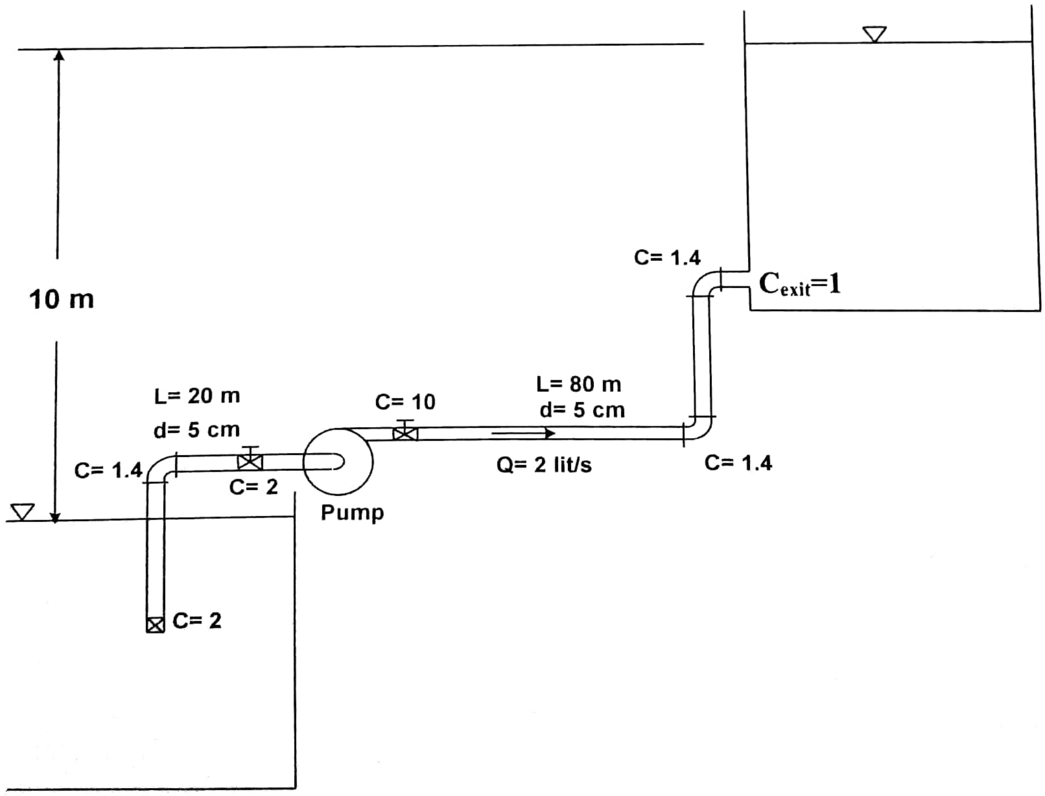
$$f = \emptyset\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

(b) A jet of water flows smoothly on to a stationary curved vane which turns it through 60°. The initial jet is 50 mm in diameter, and the velocity, which is uniform, is 30 m/s. As a result of friction, the velocity of the water leaving the surface is 27 m/s. Neglecting gravity effects, calculate the hydrodynamic force on the vane.



Question (5) [10 Marks]

Calculate the pump power for the water piping system shown in Figure. Assume pump efficiency is 70%, and *f*=0.02.



Good Luck
Dr. Ahmed Abd-Elsalam

Model answer

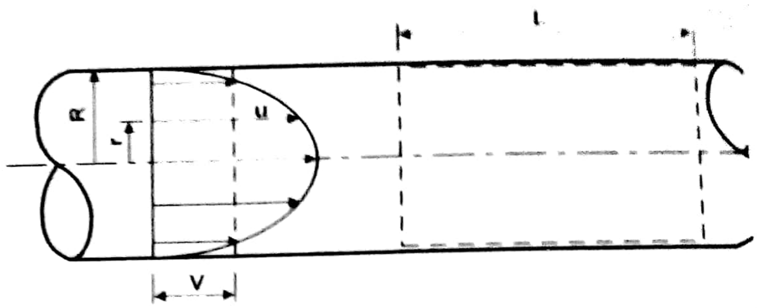
Fluid Mechanics-MPE171

Program of Biomedical Engineering

Question (1)

$$\tau = \mu \frac{du}{dy} = -\mu \frac{du}{dr}$$

$$\tau = \mu \frac{4Vr}{R^2}$$



At the pipe center line (r=0): $\tau=0$

At the wall of pipe (r=R): $\tau_w = \mu \frac{4V}{R}$

Wall shear force $F = \tau_w A = \mu \frac{4V}{R} 2\pi RL = 8\mu\pi LV$

Power loss in pipe due to viscosity

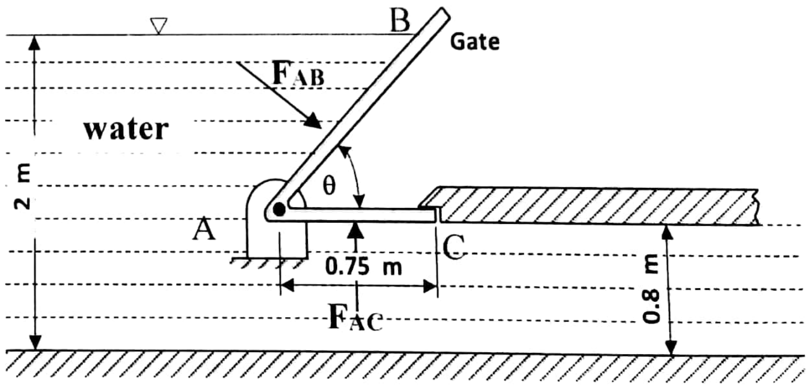
$P = FV = 8\mu\pi LV^2$

Question (2)

(a)

Gate AC:

$P_A = \gamma h = 9810 \times (2 - 0.8) = 11772\ Pa$



$A = 0.75 \times 1 = 0.75\text{m}^2$

$F_{AC} = P_A A = 8829\ N$

Gate AB:

$$A = \frac{2 - 0.8}{\sin\theta} \times 1 = \frac{2 - 0.8}{\sin\theta}$$

$$h_c = \frac{2 - 0.8}{2} = 0.6\ m$$

$$F_{AB} = \gamma Ah = 9810 \times \frac{2 - 0.8}{\sin\theta} \times 0.6 = \frac{7063.2}{\sin\theta}$$

$$y_p = y_c + \frac{I_{cc}}{Ay_c} = \frac{0.6}{sin\theta} + \frac{\frac{1}{12} \times 1 \times (\frac{2-0.8}{\sin\theta})^3}{(\frac{2-0.8}{\sin\theta})(\frac{0.6}{sin\theta})} = \frac{0.6}{sin\theta} + \frac{0.2}{sin\theta} = \frac{0.8}{sin\theta}$$

$$\sum M = 0 \text{ about A}$$

$$F_{AB} \times \left(\frac{2-0.8}{\sin\theta} - \frac{0.8}{sin\theta}\right) = F_{AC} \times \frac{0.75}{2}$$

$$F_{AB} \times \left(\frac{2-0.8}{\sin\theta} - \frac{0.8}{sin\theta}\right) = F_{AC} \times \frac{0.75}{2}$$

$$\frac{7063.2}{\sin\theta} \times \left(\frac{0.4}{\sin\theta}\right) = 8829 \times \frac{0.75}{2}$$

$$\theta = 67.48$$

(b) **Consider a wooden cylinder (sg=0.6) 1 m in diameter and 0.8 m long. Would this cylinder be stable if placed to float with its axis vertical in oil (sg=0.8)?**

$$h = \frac{\rho_b}{\rho_o} L = \frac{600}{800} \times 0.8 = 0.6\ m$$

$$BG = \frac{L}{2} - \frac{h}{2} = 0.1\ m$$

$$BM = \frac{I_{cc}}{V_s} = \frac{\frac{\pi}{64} D^4}{\frac{\pi}{4} D^2 h} = 0.104\ m$$

$BM > BG$ *the cylinder is stable*

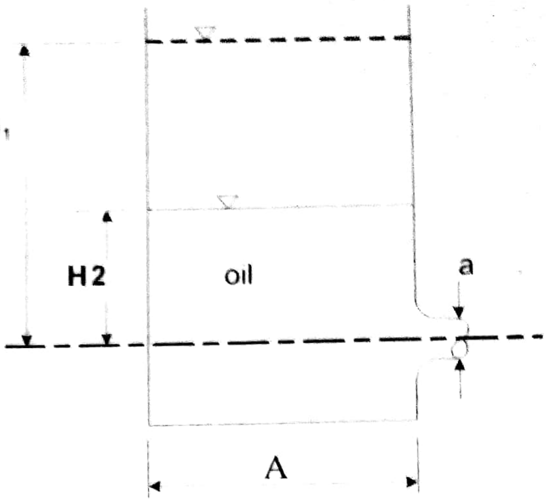
Question (3)

(a) Drive the following formula to determine the time to lower the liquid level in a tank of a constant cross section by means of an orifice.

$$t = \frac{2A}{cd\, a\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$

Solution:

(a)



$$v = \sqrt{2gh}$$

Assume that dV of liquid flows out in time dt with the liquid level falling by -dh. Then

$$dV = C_d a\sqrt{2gh}\, dt = -dh\, A$$

$$dt = -\frac{A}{C_d a}\frac{dh}{\sqrt{2gh}}$$

$$\int_0^t dt = \frac{-A}{C_d a\sqrt{2g}}\int_{H1}^{H2}\frac{dh}{\sqrt{h}}$$

The time needed for the liquid level to descend from H_1 to H_2 is

$$t = \frac{2A}{cd\, a\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$

b)

$24 = K\left(\sqrt{3} - \sqrt{2.7}\right)$, **K=270.01**

$5{\times}24 = 270.01\left(\sqrt{2.7} - \sqrt{H_2}\right)$

H_2= 1.436 m

Question (4)

(a) $f = \emptyset(V, \rho, \mu, \varepsilon, D)$

No. of groups n= 6
No. of Dimensions m= 3
No. of groups z= n-m= 3

V	LT^{-1}
ρ	ML^{-3}
μ	$ML^{-1}T^{-1}$
ε	L
D	L

Let ρ, V, D are repeating variables

$\pi_1 = f$

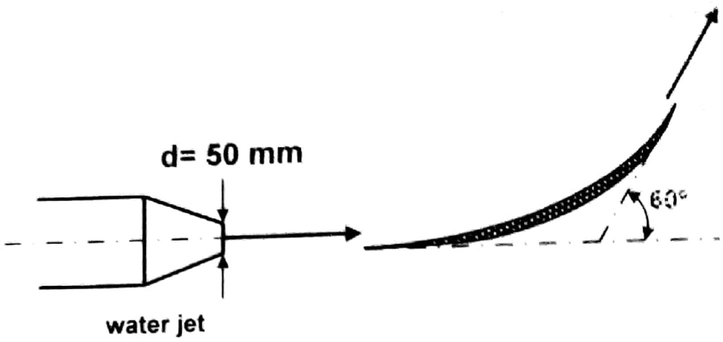
$\pi_2 = \rho^a V^b D^c \mu$ $\qquad \therefore\ \pi_2 = \frac{\rho V D}{\mu}$

$$\pi_3 = \rho^a V^b D^c \varepsilon \qquad \therefore \pi_3 = \frac{\varepsilon}{D}$$

$$\therefore \pi_1 = \varphi(\pi_3, \frac{1}{\pi_2})$$

(b)

Given: d=0.05m, V_1 =30 m/s, V_2= 27 m/s



$$m = \rho \frac{\pi}{4} d^2 V_1 = 58.875\ kg/s$$

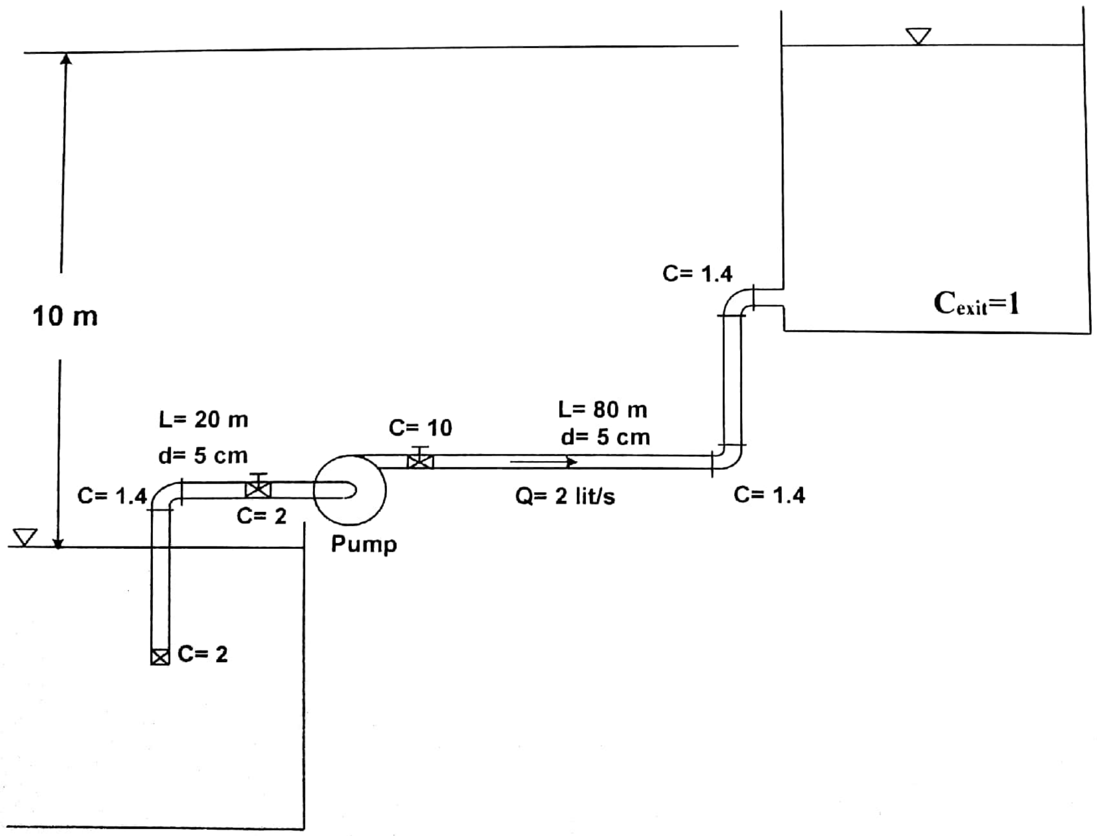
$$F_X = m(V_2 \cos(60) - V_1) = -970.7\ N$$

$$F_y = m(V_2 \sin(60) - 0) = 1376.233\ N$$

$$F_r = \sqrt{F_x^2 + F_y^2} = 1684.125\ N$$

Question (5)

Calculate the pump power for the water piping system shown in Figure. Assume pump efficiency is 70%, and f=0.02.



$$Q = \frac{\pi}{4} d^2 V$$

$$2 \times 10^{-3} = \frac{\pi}{4} 0.05^2 \times V$$

V= 1.019 m/s

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + H_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{Loss}$$

Where $P_1=P_2=0$, $V_1= V_2\cong 0$, $Z_1-Z_2=10m$

$$0 + 0 + 0 + H_p = 0 + 0 + 10 + \frac{V^2}{2g}\left[\frac{fL}{D} + 2 + 1.4 + 2 + 10 + 1.4 + 1.4 + 1\right]$$

H_p = 13.133 m

$$\eta_p = \frac{\gamma Q H_p}{Power} = \frac{9810 \times 0.002 \times 13.133}{Power} = 0.7$$

Power= 368.099 Watt