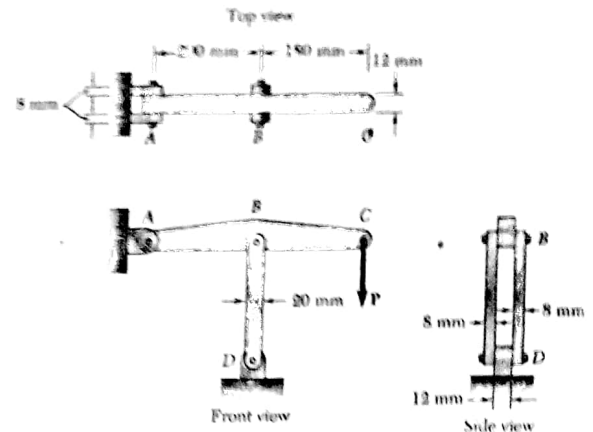


Second Midterm

Mid-Term Exam

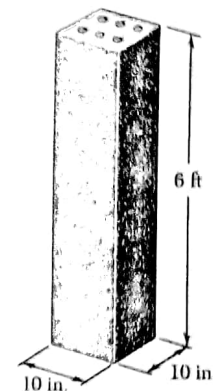
Question 1: [8 Marks]

In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load *P* if an overall factor of safety of 3.0 is desired.



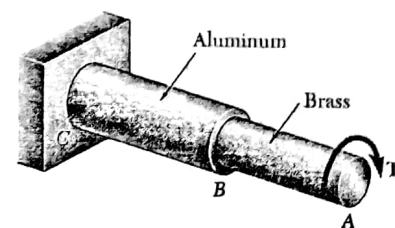
Question 2: [6 Marks]

The concrete post ($E_c = 3.6 \times 10^6$ psi and $\alpha_c = 5.5 \times 10^{-6}/^\circ\text{F}$) is reinforced with six steel bars, each of $\frac{7}{8}$ -in. diameter ($E_s = 29 \times 10^6$ psi and $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F .



Question 3: [6 Marks]

The solid rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is hollow and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (a) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (b) the largest torque that can be applied at *A*.



Best Wishes

Associate Prof. Dr. Noha Fouda

Figure

Front view Side view

PROBLEM 1

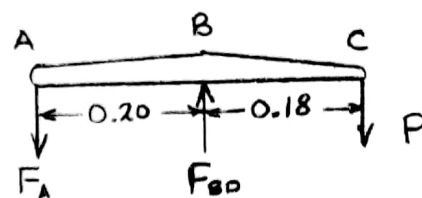
In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load *P* if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use *ABC* as free body.

$$+\circlearrowleft \Sigma M_B = 0: 0.20 F_A - 0.18 P = 0 \quad P = \frac{10}{9} F_A$$

$$+\circlearrowleft \Sigma M_A = 0: 0.20 F_{BD} - 0.38 P = 0 \quad P = \frac{10}{19} F_{BD}$$



Based on double shear in pin *A*: $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.008)^2 = 50.266 \times 10^{-6} \text{ m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_A = 3.72 \times 10^3 \text{ N}$$

Based on double shear in pins at *B* and *D*: $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.10 \times 10^{-6} \text{ m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3 \text{ N}$$

Based on compression in links *BD*: For one link, $A = (0.020)(0.008) = 160 \times 10^{-6} \text{ m}^2$

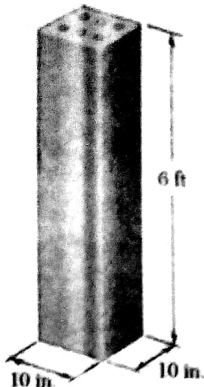
$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3 \text{ N}$$

Allowable value of *P* is smallest, $\therefore P = 3.72 \times 10^3 \text{ N}$

$P = 3.72 \text{ kN}$

6



PROBLEM 2

The concrete post ($E_c = 3.6 \times 10^6$ psi and $\alpha_c = 5.5 \times 10^{-6}/^\circ\text{F}$) is reinforced with six steel bars, each of $\frac{7}{8}$ -in. diameter ($E_s = 29 \times 10^6$ psi and $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F .

SOLUTION

$$A_s = 6 \frac{\pi}{4} d^2 = 6 \frac{\pi}{4} \left(\frac{7}{8} \right)^2 = 3.6079 \text{ in}^2$$

$$A_c = 10^2 - A_s = 10^2 - 3.6079 = 96.392 \text{ in}^2$$

Let P_c = tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c .

$$\text{Strains: } \epsilon_s = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T) \quad \epsilon_c = \frac{P_c}{E_c A_c} + \alpha_c (\Delta T)$$

$$\text{Matching: } \epsilon_c = \epsilon_s \quad \frac{P_c}{E_c A_c} + \alpha_c (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right) P_c = (\alpha_s - \alpha_c) (\Delta T)$$

$$\left[\frac{1}{(3.6 \times 10^6)(96.392)} + \frac{1}{(29 \times 10^6)(3.6079)} \right] P_c = (1.0 \times 10^{-6})(65)$$

$$P_c = 5.2254 \times 10^3 \text{ lb}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{5.2254 \times 10^3}{96.392} = 54.210 \text{ psi}$$

$$\sigma_c = 54.2 \text{ psi}$$

$$\sigma_s = -\frac{P_c}{A_s} = -\frac{5.2254 \times 10^3}{3.6079} = -1448.32 \text{ psi}$$

$$\sigma_s = -1.448 \text{ ksi}$$

Q3

6

Solid BC

$$\tau = \frac{Tc}{J}$$

$$J = \frac{\pi}{2} c^4$$

$$\tau_{all} = 25 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2} d = 0.015 \text{ m}$$

$$T_{all} = \frac{\pi}{2} c^3 \tau_{all} = \frac{\pi}{2} (0.015)^3 (25 \times 10^6) \\ = 132.536 \text{ N}\cdot\text{m} \quad 2$$

Hollow AB

$$\tau_{all} = 50 \times 10^6 \text{ Pa}$$

$$T_{all} = 132.536 \text{ N}\cdot\text{m}$$

$$c_2 = \frac{1}{2} d_2 = \frac{1}{2} (0.025) = 0.0125 \text{ m}$$

$$c_1 = ??$$

$$T_{all} = \frac{J T_{all}}{c_2} = \frac{\pi}{2} (c_2^4 - c_1^4) \frac{\tau_{all}}{c_2} \quad 2$$

$$c_1^4 = c_2^4 - \frac{2 T_{all} c_2}{\pi \tau_{all}}$$

$$= 0.0125^4 - \frac{2 (132.536) (0.0125)}{\pi (50 \times 10^6)}$$

$$c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm} \quad 1$$

$$a) \quad d_1 = 2c_1 = 15.18 \text{ mm} \quad 1$$

$$b) \quad \text{allowable torque} \quad T_{all} = 132.5 \text{ N}\cdot\text{m}$$