Almansoura University
Faculty of Engineering
BME @ MTE Progs.

Electromagnetic Theory
Final Exam
Time : 2 Hours

Answer the following questions: (Assuming any missing data)

1) a- An infinite line charge of charge density ρl = 2nC/m lies along the x axis in free space, while two point charges of 8nC are located at (0, 0, 1) and (0,0,-1). Find E at (2, 3, -4). To what value should ρl be changed to cause E to be zero at (0, 0, 3)? **(10 degrees)**

2) A uniform volume charge density of 80 $\mu C/m^3$ is present throughout the region 8 mm < r < 10 mm. **(10 degrees)**
   a- Find the total charge inside the spherical surface r=10 mm.
   b- Find D_r at r=10 mm.
   c- If there is no charge for r> 10 mm, find D_r at r=20 mm.

3) Given a surface charge density of 8nC/m2 on the plane x=2, a line charge density of 30nc/m on line x=1,y=2, and a 1 uC point charge at P(-1,-1,2) find:
   a- VAB for points A(3,4,0) and B(4,0,1). **(10 degrees)**
   b- The work expended in moving a point charge 10uC from A to B.

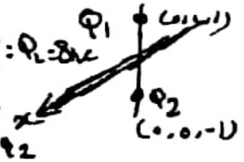
4) Region 1, defined by y > 3, is free space, while region 2, y < 3, is a dielectric material for which εr2 = 2.4. Find **E1, D2, E2,** given that **D1** = 3 ax – 4 ay + 6 az (C/m2). **(5 degrees)**

5) a- Find **H** in Cartesian components at p(2, 3, 4) if there is a current filament on the z axis carrying 8 mA in the a_z direction. **(10 degree)**
   b- Repeat if the filament is located at x = -1, y = 2.
   c- Find **H** if both filaments are present.

6) For a plane wave propagates in a lossless material, find: **(5 degrees)**
   a- Attenuation constant,
   b- Phase constant,
   c- Phase velocity,
   d- Group velocity, and
   e- Wave impedance.

Fields Model Answer

1) $\rho_L = 2\,nC/m$, $Q_1 = Q_2 = 8\,nC$



1) $\vec{E}_T = \vec{E}_{Line} + \vec{E}_{Q_1} + \vec{E}_{Q_2}$

$\vec{E}_{Q_1} = \frac{Q_1 \vec{R}_1}{4\pi\varepsilon_0 |R_1|^3}$, $\vec{E}_{Q_2} = \frac{Q_2 \vec{R}_2}{4\pi\varepsilon_0 |R_2|^3}$

$\vec{R}_1 = (2,3,-4) - (0,0,1) = 2\hat{x} + 3\hat{y} - 5\hat{z}$

$|R_1| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

$\vec{R}_2 = 2\hat{x} + 3\hat{y} - 3\hat{z}$, $|R_2| = \sqrt{22}$

$\vec{E}_L = \frac{\rho_L}{2\pi\varepsilon_0} \frac{\vec{R}_\perp}{|R_\perp|^2}$

$\vec{R}_\perp = (2,3,-4) - (2,0,0) = 3\hat{y} - 4\hat{z}$

$|R_\perp| = \sqrt{3^2 + 4^2} = 5$

$\boxed{\therefore \vec{E}_T = 2\hat{x} + 7.33\hat{y} - 9.38\hat{z}\ V/m}$

b) $\rho_L = ? \longrightarrow E_T = 0$

$\vec{R}_\perp = (0,0,3) - (0,0,0) = 3\hat{z}$, $|R_\perp| = 3$

$\vec{R}_1 = (0,0,3) - (0,0,1) = 2\hat{z}$, $|R_1| = 2$

$\vec{R}_2 = 4\hat{z}$, $|R_2| = 4$

$\therefore \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0} \frac{3\hat{z}}{9} + \frac{Q}{4\pi\varepsilon_0}\left[\frac{2\hat{z}}{8} + \frac{4\hat{z}}{64}\right] = 0$

$\therefore \frac{\rho_L}{6\pi\varepsilon_0} = \frac{-5Q}{64\pi\varepsilon_0}$

$\boxed{\therefore \rho_L = -3.75\ nC/m}$

2) a) $\rho_V \times V = Q$

$Q = \rho_V (V_2 - V_1)$

$= 80\times10^{-6} \times \frac{4}{3}\pi(10^3 - 8^3) \times 10^{-9}$

$\boxed{\therefore Q = 0.163\ nC}$

b) $\because D \times area = Q_{enc}$

$\therefore D = \frac{0.163\times10^{-9}}{4\pi(10\times10^{-3})^2} = \boxed{0.129\ nC/m^2}$

c) $D \times area = Q_{enc}$

$\therefore D = \frac{0.163\times10^{-9}}{4\pi(20\times10^{-3})^2} = \boxed{0.0324\ nC/m^2}$

3) a) $V_{AB} = V_{ABQ} + V_{ABL} + V_{ABS}$

$* V_{ABQ} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{|R_A|} - \frac{1}{|R_B|}\right]$

$|R_A| = \sqrt{45}$, $|R_B| = \sqrt{27}$

$\boxed{\therefore V_{ABQ} = -389.87\ V}$

$* V_{AB_L} = \frac{\rho_L}{2\pi\varepsilon_0} \ln\frac{|R_{\perp B}|}{|R_{\perp A}|}$

$|R_{\perp A}| = \sqrt{8}$, $|R_{\perp B}| = \sqrt{13}$

$\boxed{\therefore V_{AB_L} = 130.91\ V}$

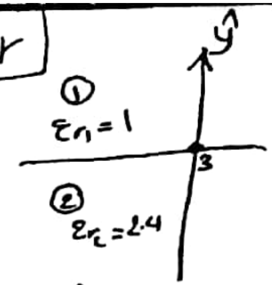
$* V_{ABS} = \frac{\rho_s}{2\varepsilon_0}(x_B - x_A)$

$\boxed{\therefore V_{ABS} = 451.76\ V}$

$\boxed{\therefore V_{AB} = 192.8\ V}$

b) $W_{AB} = Q \cdot V_{BA} = 10^{-6} \cdot -192.8 = \boxed{-192.8\ mJ}$

4)



① $\varepsilon_{r_1} = 1$

② $\varepsilon_{r_2} = 2.4$

$\hat{n} = \hat{y}$

$\hat{t} = \hat{x}\ \&\ \hat{z}$

$\vec{E}_{t_1} = \vec{E}_{t_2} = \frac{\vec{D}_{t_1}}{\varepsilon_1}$

$\therefore \vec{E}_{t_2} = \frac{3\hat{x} + 6\hat{z}}{\varepsilon_0} = \boxed{3.38\hat{x} + 6.77\hat{z}\ \times10^{11}\ V/m}$

$\vec{D}_{n_2} = \vec{D}_{n_1} = \varepsilon_2 \vec{E}_{n_2}$

$\therefore \vec{E}_{n_2} = \frac{-4\hat{y}}{\varepsilon_0 \times 2.4} = -1.88\times10^{11}\ V/m$

$\therefore \boxed{\vec{E}_1 = \frac{\vec{D}_1}{\varepsilon_0} = 3.38\hat{x} + 6.77\hat{z} - 4.5\hat{y}\ \times10^{11}\ V/m}$

$\boxed{\vec{D}_2 = 7.2\hat{x} - 4\hat{y} + 14.4\hat{z}\ C/m^2}$

$\boxed{\vec{E}_2 = 3.38\hat{x} - 1.88\hat{y} + 6.77\hat{z}\ \times10^{11}\ V/m}$

5) a) $\vec{H} = \frac{I}{2\pi\rho}\hat{\phi} = \frac{8\times10^{-3}}{2\pi\sqrt{13}}\hat{\phi} = \boxed{3.53\times10^{-4}\hat{\phi}}$

In carteisian: $\vec{H} = 3.53\times10^{-4}\left(-\frac{y}{\rho}\hat{x} + \frac{x}{\rho}\hat{y}\right)$

$\boxed{\therefore \vec{H} = -2.93\times10^{-4}\hat{x} + 1.958\times10^{-4}\hat{y}}$

b) $\vec{H} = \frac{I}{2\pi|R_\perp|}\hat{\phi}$

$\vec{R}_\perp = (2,3,4) - (-1,2,4) = 3\hat{x} + \hat{y}$

$|R_\perp| = \sqrt{10}$

$\therefore \vec{H} = \frac{8\times10^{-3}}{2\pi\sqrt{10}}\hat{\phi}$

$\boxed{\therefore \vec{H} = 4.02\times10^{-4}\hat{\phi}}$

In carteisian: $\vec{H} = 4.02\times10^{-4}(\ldots)$

$\boxed{\therefore \vec{H} = -1.27\times10^{-4}\hat{x} + 3.813\ldots}$

c) $\vec{H}_T = \vec{H}_1 + \vec{H}_2$

$\therefore \vec{H}_T = -4.2\times10^{-4}\hat{x} + 5.77\times10^{-4}\hat{y}$

$\boxed{\therefore \vec{H}_T = -0.42\hat{x} + 0.577\hat{y}\ mA/m}$

6) a) attenuation constant

$\alpha = 0$

b) phase constant

$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon}$

c) phase velocity

$v_{ph} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$

d) Group velocity

$v_G = \frac{\partial\omega}{\partial\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$

e) wave impedance

$\eta = \frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}}$

$E \perp H$

$0, \phi$