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Electromagnetic Fields
Course Code: ECE262
Spring Semester Exam.

BME Program
Level 200
Exam Date: 10-5-2018
Allowed Time: 2 Hours

Open-Sheet

Attempt all questions. Assume any missed data. Full mark is 50.

Q.1.a) *State Coulomb's law.* If the volume charge density varies with radius such that $\rho_v = \frac{3}{r} nc/m^3$. Find the total charge lying within the region bounded by the sphere $r \le 3$ and the cone $0 \le \theta \le \pi/3$. $13.5\pi = 42.4nC$ **[4 Marks]**

Q.1.b) A charge Q is located at (-1,0,0) and a charge 2Q is located at (2,0,0). Find the point in space where E=0? $(0.2426, 0, 0)$ **[5 Marks]**

Q.1.c) A uniform line charge of $\rho_L = 4\pi \, nC/m$ lies along the y axis, while uniform surface charge densities of + 0.2 and -0.2 nC/m2 exist on the planes z=6 m and z=-8 m, respectively. Find E at the point P (1,0,0). $225.9 a_x - 22.5 \hat{a}_z$ **[5 Marks]**

Q.2.a) Verify divergence theorem for the electric flux density vector $\overline{D} = 8x^2y^2\hat{a}_x + 4x^2y\hat{a}_y$ leaving the surface of the cube: -1≤x≤1, -1≤y≤1, -1≤z≤1 $\frac{32}{3}$ **[7 Marks]**

Q.2.b) Given $\overline{E} = 5y\hat{a}_x + 5x\hat{a}_y - 2\hat{a}_z V/m$. Determine the work involved in carrying a charge of 2C from (0,-2,8) to (5,3,23) along the straight line joining the two points. $-9cJ$ **[7 Marks]**

$V = 4V$
$\bar{E} = -2\hat{a}_\rho$
$\rho_v = -8.854 * 10^{-12}$
$\phi = 0$

Q.3.a) The potential varies as $V = 2\rho\hat{a}_\rho$ in cylindrical coordinates. Find the potential *V*, the electric field, and the volume charge density ρ_V, at *P(2,0°,2)*. Find the equipotential surface and the equation of streamlines through P. Does *V* satisfy Laplace equation? NO **[7 Marks]**

Q.3.b) Use *Laplace* equation, in spherical coordinates, to determine the capacitance of a spherical capacitor with inner conductor of radius *'a'* and potential $V = V_o$ and outer conductor of radius *'b'* and potential *V=0.* $C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$ **[7 Marks]**

Q.5.a) A coaxial cable has inner conductor of radius 'a' and carries a current of 1A and outer conductor of inner radius 'b' and outer radius 'c' and carries a current -1A. Use Ampere's law to derive expressions for the magnetic field everywhere. Sketch the resulting field. **[7 Marks]**

Q.5.b) Write down Maxwell's equations for steady electric and magnetic fields in both point form and integral forms. If magnetic field intensity is given as $\overline{H} = 2yz\hat{a}_x + 3xz\hat{a}_y + 2xy\hat{a}_z$, find the current density 'J'. $J = -x\hat{a}_x + z\hat{a}_z$ **[7 Marks]**

My best wishes to all of you! ***Assoc. Prof. Hossam El-Din Moustafa***

Model Answer — Electromagnetics

Spring 2018

$\boxed{Q_1}$ هو القوة المتبادلة بين شحنتين حيث يتناسب طردياً مع حاصل ضرب الشحنتين وعكسياً مع مربع المسافة بينهم وتكون القوة في اتجاه متجه $\vec{r}_{12}$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon|r_{12}|^2}\,\hat{a}_{r_{12}}$$

$$\epsilon = 8.854\times10^{-12}\ \frac{F}{m}$$

$$\rho_v = \frac{3}{r}\ \frac{nC}{m^3} \qquad r \le 3$$

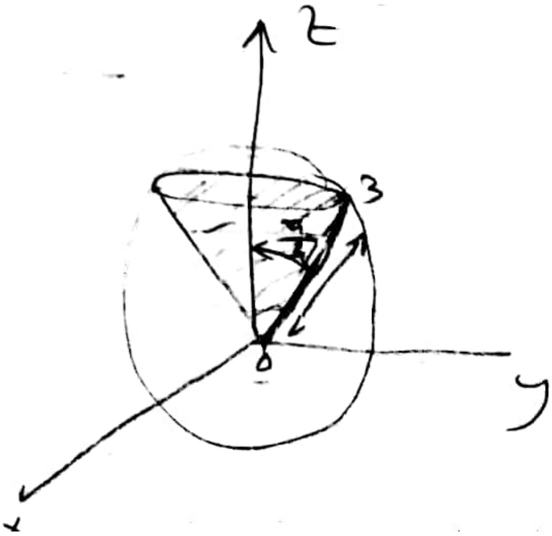
cone $0 \le \theta \le \frac{\pi}{3}$

$$Q = \iiint \rho_v\, dv \qquad dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$Q = \int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{3} \frac{3}{\cancel{r}}\, r^{\cancel{2}} \sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{27}{\cancel{2}}(\cancel{2}\pi)\left(\frac{1}{2}\right) = \frac{27\pi}{2}\ nC$$

$$= 42.41\ nC$$

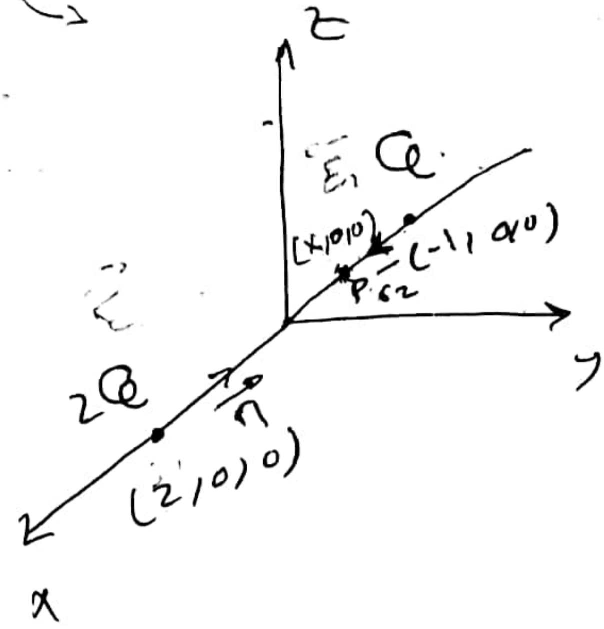


$+2Q \; \} \; E=0 \; Q$

$\boxed{Q_1 \, b}$

$(2,0,0)$ $\quad (x,0,0)$

$2Q \xrightarrow{\vec{r}_1} P$



شحنتین زه بعضی مجال = صفر داخلها

$$\vec{r}_1 = x-2 \quad \hat{x}$$

$$|r_1| = x-2$$

$$\vec{E}_1 = \frac{2Q}{4\pi\epsilon (x-2)^2}\hat{x}$$

$$\vec{r}_2 = x+1 \quad \hat{x}$$

$$|r_2| = x+1$$

$$\vec{E}_2 = \frac{Q}{4\pi\epsilon (x+1)^2}\hat{x}$$

$$\vec{E}_1 + \vec{E}_2 = 0$$

$$\frac{2\cancel{Q}}{4\cancel{\pi\epsilon}(x-2)^2} = -\frac{\cancel{Q}}{4\cancel{\pi\epsilon}(x+1)^2}$$

$(\pm\ r_{2i}, 0, 0)$

$$\frac{2}{(x-2)^2} = \frac{-1}{(x+1)^2}$$

$x^2 - 4x + 4$

$x^2 + 2x + 1$

$$2(x^2+2x+1) = -(x^2-4x+4)$$

$$2\cancel{x^2}+4x+2 = -\cancel{x^2}+4x-4$$

$$3x^2+2+4=0$$

$$\cancel{3x^2=-6}$$

$$\cancel{x^2=\frac{-6}{3}}$$

Solution

$-\hat{x} \rightarrow$ $\quad \leftarrow +\hat{x}$

$$2Q \xrightarrow{r_1} P \xleftarrow{r_2} Q$$

$(x,0,0)$

$(2,0,0)$ $\quad (-1,0,0)$

$(-1-x)\hat{x}$

$$\vec{r}_1 = (x-2)\,\hat{x}$$

$$|r_1| = x-2$$

$$\bar{E}_1 = \frac{-2Q}{4\pi\epsilon\,(x-2)^2}\,\hat{x}$$

$$\bar{r}_2 = x+1\ \hat{x}$$

$$|r_2| = x+1$$

$$\vec{E}_2 = \frac{Q}{4\pi\epsilon(x+1)^2}\,\hat{x}$$

$$\frac{2}{(x-2)^2} = \frac{-1}{(-1-x)^2}$$

$$\bar{E}_1 + \bar{E}_2 = 0$$

$$\frac{-2\cancel{Q}}{\cancel{4\pi\epsilon}(x-2)^2} + \frac{\cancel{Q}}{\cancel{4\pi\epsilon}(x+1)^2} = 0$$

$$2(1+2x+x^2) = x^2-4x+4$$

$$2+4x+2x^2 = x^2-4x+4$$

$$x^2+8x+2-4=0$$

$$\frac{1}{(x+1)^2} = \frac{2}{(x-2)^2}$$

$\therefore P(0.2426, 0, 0)$

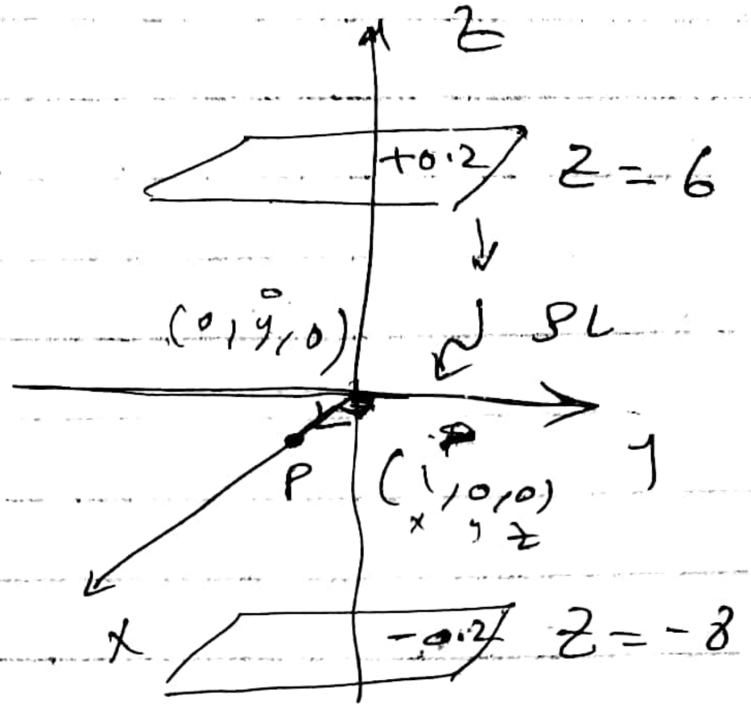
$$2[x^2+2x+1] = x^2-4x+4$$

$$x^2+8x-2=0$$

$x \rightarrow 0.2426$

$x \rightarrow -8.242$ مرفوضة

Q1c $\rho_L = 4\pi\, nC/m$



$\bar{\rho} = 1\,\hat{x}$

$|\rho| = 1$

$$\vec{E}_{line} = \frac{\rho_L}{2\pi\epsilon\rho^2}\bar{\rho}$$

$$= \frac{4\pi\times10^{-9}}{2\pi\epsilon\times(1)^2}\hat{x}$$

$$\vec{E}_{line} = 225.9\,\hat{x}$$

$$\vec{E}_{sh_1} = \frac{\rho_s}{2\epsilon}\hat{a}_n$$

$$= \frac{0.2\times10^{-9}}{2\times8.854\times10^{-12}} -\hat{z}$$

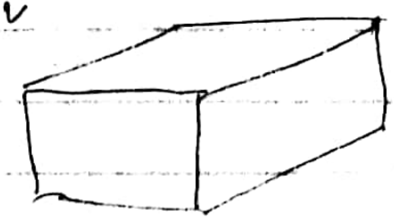
$$\vec{E}_{sh_1} = -11.29\,\hat{z}$$

$$\vec{E}_{sh_2} = \frac{-0.2\times10^{-9}}{2\times8.854\times10^{-12}}\hat{z} = -11.29\,\hat{z}$$

$$\vec{E}_T = \vec{E}_{line} + \vec{E}_{sh_1} + \vec{E}_{sh_2} = 225.9\hat{x} - 22.58\hat{z}$$

$\boxed{Q_2\ a)}$ $\bar{D} = 8x^2y^2\,\hat{a}_x + 4x^2y\,\hat{a}_y$

$$\iint_S \bar{D}\cdot ds = \iiint_v (\bar{\nabla}\cdot\bar{D})\,dv$$



R.H.S :

$$\bar{\nabla}\cdot\bar{D} = 16xy^2 + 4x^2$$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 16xy^2 + 4x^2 \; dx\,dy\,dz$$

$$\frac{16x^2}{2}y^2 + \frac{4x^3}{3}\Big|_{-1}^{1}$$

$$\cancel{8y^2} + \frac{4}{3} - \left(\cancel{8y^2} - \frac{4}{3}\right)$$

$$\int_{-1}^{1}\int_{-1}^{1} \frac{4}{3} + \frac{4}{3}\; dy\,dz$$

$$\frac{8}{3}(2)(2) = \boxed{\frac{32}{3}}$$

L.H.S

$x = 1$

$$\int_{-1}^{1}\int_{-1}^{1} 8x^2y^2 \; dy\,dz$$

$$\frac{8y^3}{3}x^2\Big|_{-1}^{1}(2) = x^2\left[\frac{8}{3} - \left(\frac{-8}{3}\right)\right](2) = \frac{16(2)}{3}$$

$$= \frac{32}{3}$$

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$x = -1$

$$\int_{-1}^{1}\int_{-1}^{1} 8x^2y^2 \, dy\, dz$$

$$= -\frac{8x^2y^3}{3}\Big|_{-1}^{1} (2)$$

$$= -\frac{8}{3}x^2 (2)(2) = \frac{-32}{3}$$

$y = 1$ $+\hat{a}_y$

$$\int_{-1}^{1}\int_{-1}^{1} 4x^2y \, dx\, dz$$

$$y\,\frac{4x^3}{3}\Big|_{-1}^{1} (2)$$

$$\frac{8}{3}y\,(1-(-1)) = \frac{16}{3}$$

$y = -1$ $-\hat{a}_y$

$$\int_{-1}^{1}\int_{-1}^{1} 4x^2y \, dx\, dz$$

$$-\frac{4x^3}{3}\Big|_{-1}^{1} y\,(2) = -\frac{8}{3}(-1)\left[(1-(-1))\right]$$

$$= \frac{16}{3}$$

$\therefore \boxed{L.H.S = \frac{32}{3}}$

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$\boxed{Q_2 - b}$ $\vec{E} = 5y\,\hat{a}_x + 5x\,\hat{a}_y - 2\hat{a}_z$

$Q = 2C$

من $\underset{1}{(0, -2, 8)} \longrightarrow \underset{2}{(5, 3, 23)}$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 0}{5 - 0} = \frac{y + 2}{3 - (-2)}$$

$B\,(0, -2, 8)$

$$\frac{x}{\cancel{5}} = \frac{y + 2}{\cancel{5}}$$

$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

$x = y + 2$

$dx = dy$

$$W_{AB} = -Q \int_B^A \vec{E} \cdot \vec{dl}$$

$$= -2 \left[\int_{-2}^{3} 5y\,dy + \int_{0}^{5} 5x\,dx - 2\int_{8}^{23} dz \right]$$

$$= -2 \left[\frac{25}{2} + \frac{125}{2} - 30 \right] = -90\,J$$

$\boxed{Q3-9}$ $V = \underline{2\rho}$

$p(2, 0^\circ, 2)$
$\quad\rho \quad \phi \quad z$

$V = 2(2) = 4 \text{ Volt}$

$\vec{E} = -\nabla V$

$$\boxed{\vec{E} = -2\, a_\rho}$$

$-\frac{\rho_v}{\epsilon} = \nabla^2 v$ Poisson equation

$$\nabla^2 v = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial v}{\partial\rho}\right)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho(2))$$

$$\nabla^2 v = \frac{2}{\rho} = -\frac{\rho_v}{\epsilon}$$

at $\boxed{\rho = 2}$

$$\frac{2}{2} = \frac{-\rho_v}{\epsilon} \Rightarrow \rho_v = -\epsilon$$

$$\rho_v = -8.854 \times 10^{-12} \frac{C}{m^3}$$

$$\rho_v = -8.854 \frac{pC}{m^3}$$

$V = \text{Constant}$

$A = 2\rho$

$A = 2(2) \Rightarrow \boxed{A = 4}$ equipotential

Streamline

$\vec{E} = -\nabla V = -2\hat{a}_\rho + 0 + 0$

$$\frac{\partial \rho}{E_\rho} = \frac{\rho \partial \phi}{E_\phi} = \frac{\partial z}{E_z}$$

$$\frac{\partial \rho}{-2} = \frac{\rho \partial \phi}{0}$$

$$\int \frac{\partial \rho}{-2\rho} = \int \frac{\partial \phi}{0}$$

$-\frac{1}{2}\ln\rho = \frac{\phi}{0} + C$ No, Not satisfy as $\boxed{\nabla^2 V \neq 0}$

$-\frac{1}{2}\ln\rho = \infty$

$\ln\rho = -\infty$

$\rho = e^{-\infty}$

$\boxed{\rho = 1}$

$$\int \frac{\partial \rho}{-2\rho} = \int \frac{\partial \phi}{0}$$

$$-\frac{1}{2}\ln\rho = \frac{\phi}{0} + C$$

$-\frac{1}{2}\ln\rho = \infty$

$\ln\rho = -\infty$

$\rho = e^{-\infty}$

$\boxed{\rho = 1}$

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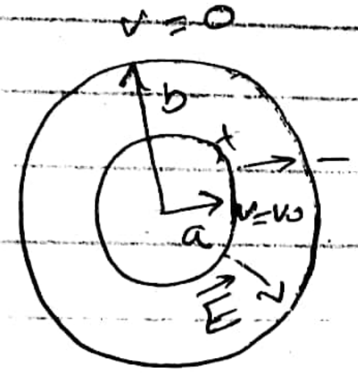
$\boxed{Q_3 - b}$



$V = V_0$ at $r = a$

$V = 0$ at $r = b$

$$\nabla^2 V = 0$$

$$\cancel{\frac{1}{r^2}} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = A$$

$$\boxed{\frac{\partial V}{\partial r} = \frac{A}{r^2}}$$

r^{-2+1}

$-\frac{1}{r}$

$$V = -\frac{A}{r} + B$$

$$V_0 = -\frac{A}{a} + \cancel{B}$$

$$0 = +\frac{A}{b} \mp \cancel{B}$$

$$V_0 = -A\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\boxed{A = \frac{-V_0}{\frac{1}{a} - \frac{1}{b}}}$$

$$0 = \frac{-A}{b} + B$$

$$B = +\frac{A}{b}$$

$$B = \frac{-V_0/b}{\frac{1}{a} - \frac{1}{b}}$$

$$\boxed{V = \frac{V_0/r}{\frac{1}{a} - \frac{1}{b}} - \frac{V_0/b}{\frac{1}{a} - \frac{1}{b}}}$$

$$\vec{E} = -\nabla V$$

$$= -\frac{\partial V}{\partial r}\hat{a}_r$$

$$\vec{E} = -\frac{A}{r^2}\hat{a}_r$$

$$\vec{E} = +\frac{V_0/r^2}{\frac{1}{a} - \frac{1}{b}}\hat{a}_r$$

$$\vec{D} = \epsilon\vec{E} \Rightarrow \vec{D} = \frac{\epsilon V_0/r^2}{\frac{1}{a} - \frac{1}{b}}\hat{a}_r$$

$$\rho_s = \frac{\epsilon V_0 / r^2}{\frac{1}{a} - \frac{1}{b}}$$

$$Q = \iint \rho_s \, ds$$

$$Q = \int_0^{2\pi}\int_0^{\pi} \frac{\epsilon V_0 / \cancel{r^2}}{\frac{1}{a} - \frac{1}{b}} \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$Q = \frac{\epsilon V_0 (2)(2\pi)}{\frac{1}{a} - \frac{1}{b}}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}}$$

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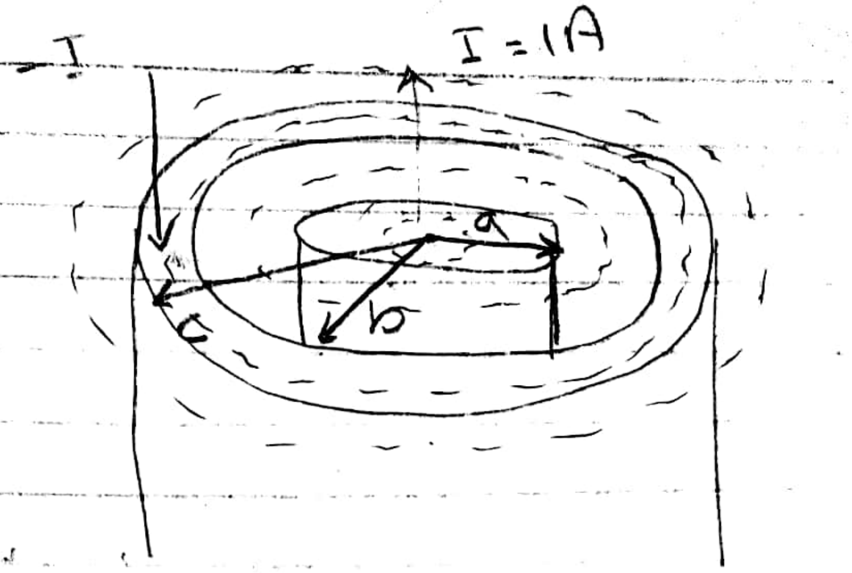
$\boxed{5a}$



a = 2
b = 4
c = 6

$\boxed{1}$ at $\rho < a$

$I \longrightarrow \pi a^2$

$I' \longrightarrow \pi \rho^2$

$$I' = \frac{I \rho^2}{a^2}$$

$$\oint \bar{H} \cdot dl = I_{enclosed}$$

$$H \, 2\pi\rho = \frac{I \rho^2}{a^2}$$

$$\vec{H} = \frac{I \rho}{2\pi a^2} \hat{\phi}$$

at I = 1A

$$\vec{H} = \frac{\rho}{2\pi a^2} \hat{\phi}$$

$\boxed{2}$ at $a < \rho < b$

$$H \, 2\pi\rho = I$$

$$\vec{H} = \frac{I}{2\pi\rho} \hat{\phi} \Rightarrow \vec{H} = \frac{1}{2\pi\rho} \hat{\phi}$$

$\boxed{3}$ at $b < \rho < c$

$I\frac{c^2-\rho^2}{c^2-b^2}$

$H\, 2\pi\rho = I - I'$ $\qquad \vec{H} = \frac{}{2\pi\rho}$

$I \rightarrow \pi\,(c^2 - b^2)$

$I' \rightarrow \pi\,(\rho^2 - b^2)$

$$I' = I\,\frac{(\rho^2 - b^2)}{(c^2 - b^2)}$$

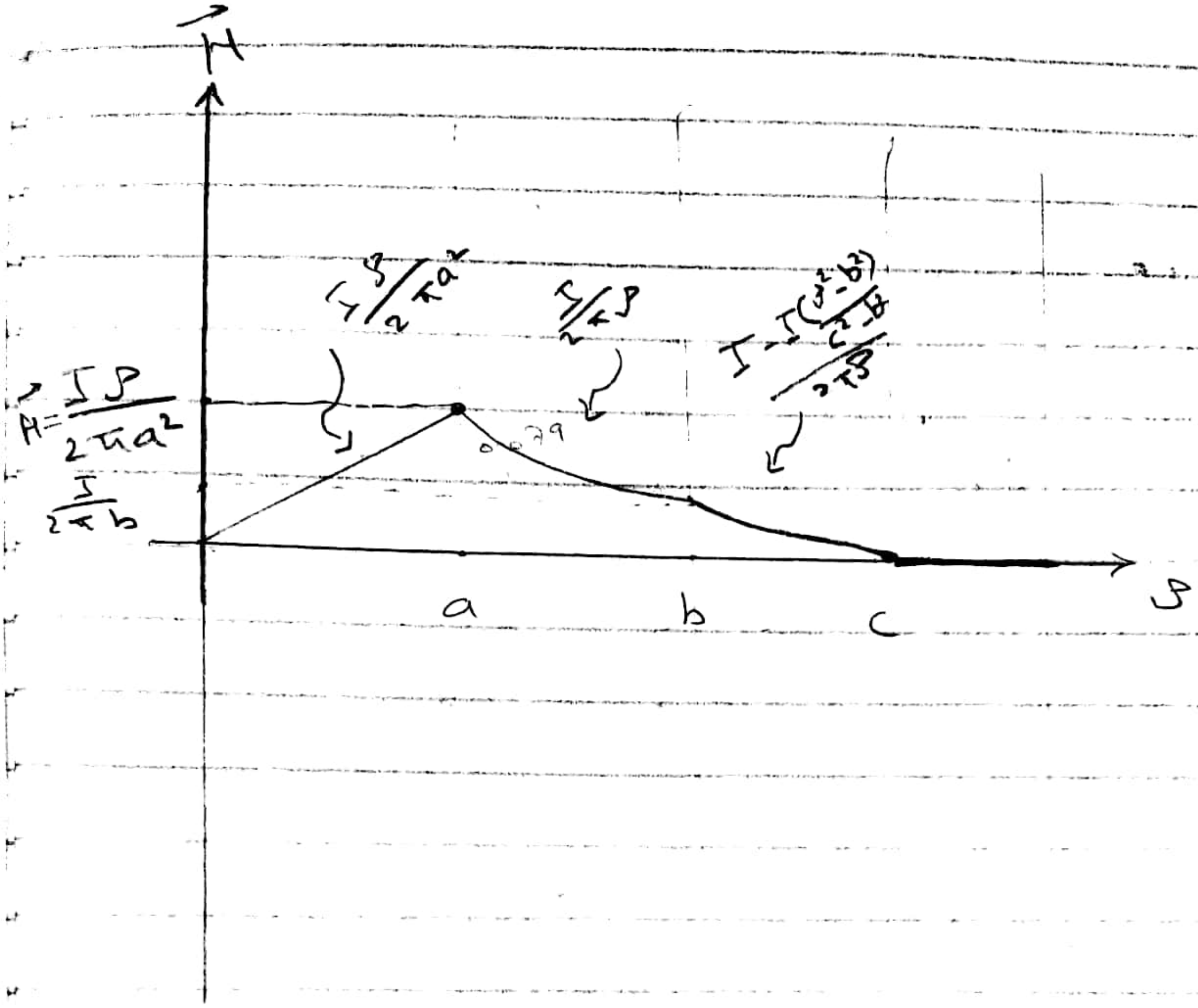
$$H\, 2\pi\rho = I - I\,\frac{(\rho^2 - b^2)}{c^2 - b^2)}$$

$$\vec{H} = \frac{I - I\,\frac{(\rho^2 - b^2)}{c^2 - b^2}}{2\pi\rho}\,\hat{\phi}$$

$$\vec{H} = \frac{I\left(\frac{c^2-\rho^2}{c^2-b^2}\right)}{2\pi\rho}\,\hat{\phi}$$

$\boxed{4}$ at $\rho > c$

$$\vec{H} = 0$$



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Q5.b

Integral	Point
$Q = \iint_S \bar{D} \cdot ds = \iiint_V \rho_v dv$	$\bar{\nabla} \cdot \bar{D} = \rho_v$ Gauss law (electricity)
$\oint \vec{E} \cdot \bar{dl} = 0$	$\bar{\nabla} \times \bar{E} = 0$ electrostatic electric field.
$I = \oint \bar{H} \cdot \bar{dl} = \iint_S \bar{J} \cdot ds$	$\bar{\nabla} \times \bar{H} = \bar{J}$ Ampere's law
$\oiint_S \bar{B} \cdot \bar{ds} = 0$	$\bar{\nabla} \cdot \bar{B} = 0$ Gauss law Magnetism

$$\vec{H} = \underbrace{2yz}_{H_x} \hat{a}_x + \underbrace{3xz}_{H_y} \hat{a}_y + \underbrace{2xy}_{H_z} \hat{a}_z$$

$$\bar{J} = \bar{\nabla} \times \bar{H}$$

$H_z = 2xy$

$H_y = 3xz$

$H_x = 2yz$

$$\bar{\nabla} \times \bar{H} = \left(\frac{\partial 2xy}{\partial y} - \frac{\partial 3xz}{\partial z} \right) \hat{a}_x$$

$$+ \left(\frac{\partial 2yz}{\partial z} - \frac{\partial 2xy}{\partial x} \right) \hat{a}_y$$

$$+ \left(\frac{\partial 3xz}{\partial x} - \frac{\partial 2yz}{\partial y} \right) \hat{a}_z$$

$$= (2x - 3x)\, \hat{a}_x + (2y - 2y)\, \hat{a}_y$$

$$+ (3z - 2z)\, \hat{a}_z$$

$$-x\, \hat{a}_x + z\, \hat{a}_z$$

$$\boxed{\bar{\nabla} \times H = -x\, \hat{a}_x + z\, \hat{a}_z} = \bar{J}$$