Model Answer

Electromagnetic Fields
Course Code: ECE262
Spring Semester
Mid-term exam

BME Program
Level 200
Exam Date: 30-3-2019
Allowed Time: 1 Hour

Open-Sheet Exam.

Answer as many questions as you can. Assume any missed data. Full mark is 20.

Q.1) A line charge of $\rho_L = 2nC/m$ along the y-axis and a uniform surface charge density of *0.5 nC/m²* exists on the plane *x = 4 m*. Find $\overline{E}$ at the point P (1,2,3). **[5 Marks]**

Q.2) A charge Q is located at (-1,0,0) and a charge 2Q is located at (2,0,0). Find the point in space where E=0? **[5 Marks]**

Q.3) A charge 2Q is located inside a conducting spherical shell with inner radius 3 cm, and outer radius 4 cm which carries a charge –Q. Determine and sketch the electric field everywhere versus 'r'. Sketch the electric field lines. **[5 Marks]**

Q.4) Given that $\overline{D} = 0.2\rho^2 \overline{a}_\rho$ C/m² within the cylinder $\rho = 4$ and $-5 \le z \le 5$, find the volume charge density ρ_V. Verify divergence theorem for the volume enclosed by this cylinder. **[5 Marks]**

Q.5) *Define equi-potential surface.* Derive an expression for the electric potential of a circular *ring* of radius *'a'* located in the *x-y* plane, at a point *(0,0,z)*. Consider the uniform linear charge density of the ring to be 'ρ_L'. Use the result to get the electric field at the same point. **[5 Marks]**

My best wishes to all of you!

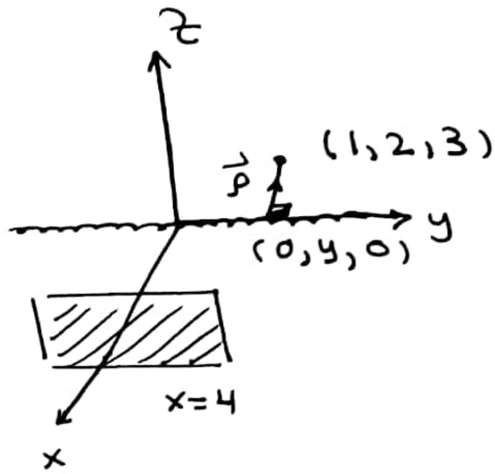
Assoc. Prof. Hossam El-Din Moustafa

$\boxed{Q1}$ given $\rho_L = 2nC/m \rightarrow y\text{-axis}$

$\rho_S = 0.5nC/m \rightarrow x = 4$

Find $\vec{E}$ at $(1,2,3)$

$$\vec{E}_T = \vec{E}_L + \vec{E}_s$$



$$* \quad \vec{E}_L = \frac{\rho_L}{2\pi\epsilon} \cdot \frac{\vec{\rho}}{|\rho|^2}$$

$$\vec{\rho} = (1,2,3) - (0,2,0)$$

$$\vec{\rho} = 1\hat{x} + 3\hat{z}$$

$$|\rho| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\vec{E}_L = \frac{2*10^{-9}}{2\pi * 8.85*10^{-12}} \cdot \frac{1\hat{x} + 3\hat{z}}{(\sqrt{10})^2}$$

$$\therefore \quad \vec{E}_L = 3.597\hat{x} + 10.791\hat{z}$$

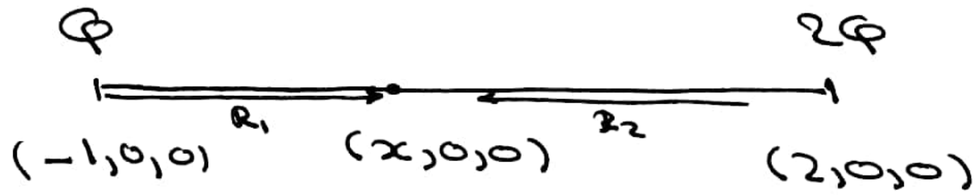
$$* \quad \vec{E}_s = \frac{\rho_s}{2\epsilon} \cdot \hat{n} \quad , \quad \hat{n} = -\hat{x}$$

$$= -\frac{0.5*10^{-9}}{2*8.85*10^{-12}} \hat{x}$$

$$\therefore \quad \vec{E}_s = -28.249\hat{x}$$

$$\vec{E}_T = -24.652\hat{x} + 10.791\hat{z} \ \ V/m$$

Q2



$$\vec{E}_T = \vec{E}_1 + \vec{E}_2 = \frac{Q}{4\pi\epsilon} \cdot \frac{\vec{R}_1}{|R_1|^3} + \frac{2Q}{4\pi\epsilon} \cdot \frac{\vec{R}_2}{|R_2|^3}$$

$$\vec{R}_1 = (x+1)\,\hat{x} \quad , \quad |R_1| = x+1$$

$$\vec{R}_2 = (x-2)\,\hat{x} \quad , \quad |R_2| = x-2$$

$$0 = \frac{Q}{4\pi\epsilon} \cdot \frac{\cancel{(x+1)}\,\hat{x}}{(x+1)^{2}} + \frac{2Q}{4\pi\epsilon} \cdot \frac{\cancel{(x-2)}\,\hat{x}}{(x-2)^{2}}$$

divide both sides by $\frac{Q}{4\pi\epsilon}$

$$0 = \frac{1}{(x+1)^2} + \frac{2}{(x-2)^2}$$

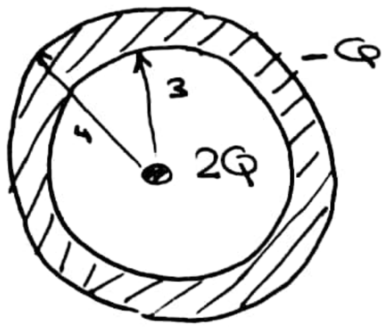
$$\frac{-1}{x^2+2x+1} = \frac{2}{x^2-4x+4}$$

$$2x^2+4x+2 = -x^2+4x-4$$

$$3x^2+6=0 \qquad x = \pm\sqrt{2}\,j$$

$\because$ imaginary $\therefore \Rightarrow$ no point where $\vec{E}=0$

Q3



gaussian surface

sphere

$\therefore Q_{enc} = D_r \, 4\pi r^2$

(1) at $r < 3$ Cm :

$$2Q = D_r \, 4\pi r^2$$

$$D_r = \frac{2Q}{4\pi r^2} \Rightarrow \vec{D} = \frac{Q}{2\pi r^2} \hat{r}$$

$$\vec{D} = \varepsilon \vec{E} \quad \therefore \Rightarrow \boxed{\vec{E} = \frac{Q}{2\pi \epsilon r^2} \hat{r}}$$

(2) at $3 < r < 4$:

$\because$ Conducting shell $\therefore \Rightarrow \boxed{\vec{E} = 0}$

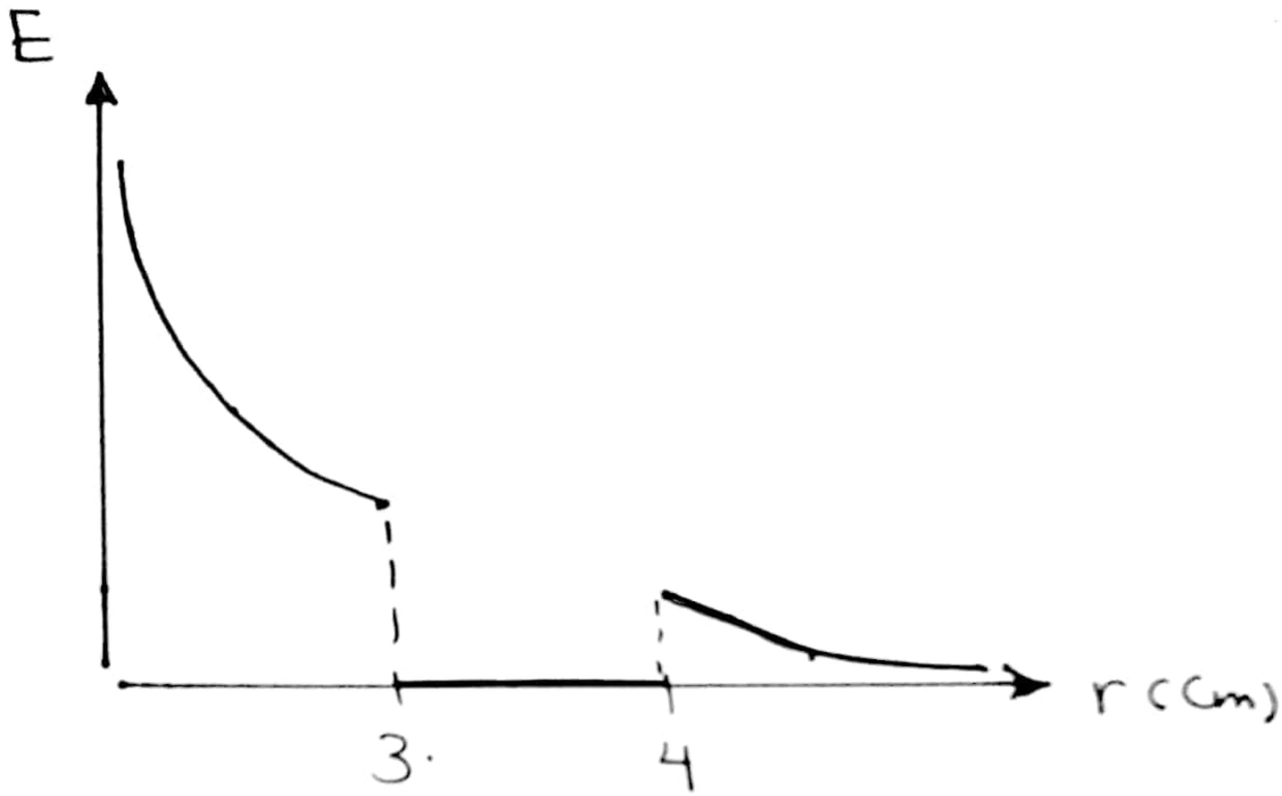
(3) at $r > 4$ Cm :

$$Q_{enc} = D_r \, 4\pi r^2$$

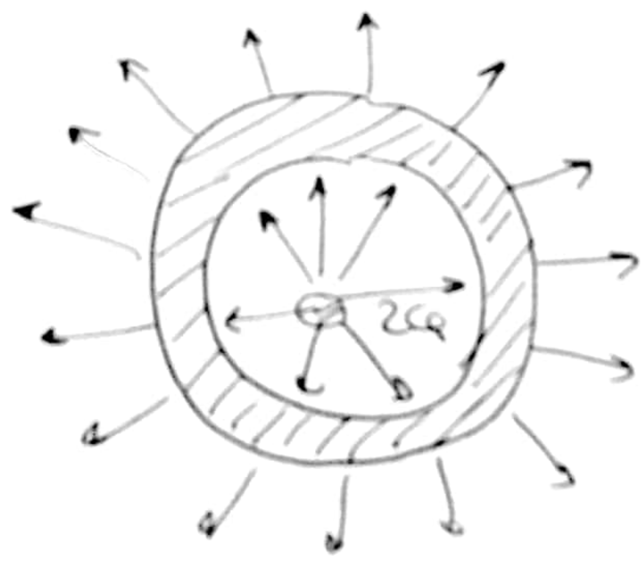
$$2Q - Q = D_r \, 4\pi r^2$$

$$D_r = \frac{Q}{4\pi r^2} \rightarrow \vec{D} = \frac{Q}{4\pi r^2} \hat{r}$$

$$\vec{D} = \varepsilon \vec{E} \quad \therefore \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi \epsilon r^2} \hat{r}}$$



Electric field lines



$\boxed{Q4}$ $\vec{D} = 0.2\rho^2 \hat{\rho}$ (5)

Region $\rho = 4$, $-5 \leq z \leq 5$

- Find ρ_v
- Verify Divergence Theorem

Sol :-

$$\nabla \cdot \vec{D} = \rho_v$$

in Cylindrical

$$\frac{1}{\rho} \frac{d}{d\rho}(\rho D_\rho) = \rho_v$$

$$\frac{1}{\rho} \frac{d}{d\rho} \rho . 0.2\rho^2$$

$$\frac{1}{\rho} \frac{d}{d\rho} 0.2\rho^3$$

$$\frac{1}{\cancel{\rho}} 0.6\rho^{\cancel{2}}$$

$$\boxed{\rho_v = 0.6\rho \ C/m^3}$$

at $\rho = 4$

$$\therefore \rho_v = 2.4 \ C/m^3$$

(1) using Maxwell equation

$$Q = \iiint \rho_v \, dv$$

$$= \int_{-5}^{5}\int_{0}^{2\pi}\int_{0}^{4} 0.6\rho \; \rho \, d\rho \, d\phi \, dz$$

$$= 0.6 \int_{0}^{4} \rho^2 \, d\rho \int_{0}^{2\pi} d\phi \int_{-5}^{5} dz$$

$$= \frac{0.6}{3} \rho^3 \Big|_{0}^{4} \cdot \phi \Big|_{0}^{2\pi} \cdot z \Big|_{-5}^{5}$$

$$= 0.2\,(64-0)\cdot(2\pi-0)\cdot(5-(-5))$$

$\therefore$ $Q = 256\pi$ Coloumb $\longrightarrow$ R.H.S

(2) using Gauss law

$$Q = \int \vec{D} \cdot d\vec{S}$$

$$dS_\rho = \rho \, d\phi \, dz \; \hat{\rho}$$

$$Q = \iint 0.2\rho^2 \, \hat{\rho} \cdot \rho \, d\phi \, dz \; \hat{\rho}$$

$$\hat{\rho} \cdot \hat{\rho} = 1$$

$$Q = \int_{-5}^{5}\int_{0}^{2\pi} 0.2\,\rho^3 \, d\phi \, dz$$

$$= 0.2\rho^3 \int_{0}^{2\pi} d\phi \int_{-5}^{5} dz$$

$$= 0.2(4)^3 * 2\pi * 10$$

$\therefore$ $Q = 256\pi$ Coloumb $\longrightarrow$ L.H.S

$\therefore$ L.H.S = R.H.S = 256π C ✓

Q5

* def:

هو عبارة عن سطح أى نقطة عليه يكون لها نفس الجهد
بمعنى أننا لا نحتاج لبذل أى شغل لنقل شحنة على هذا
السطح من نقطة إلى أخرى

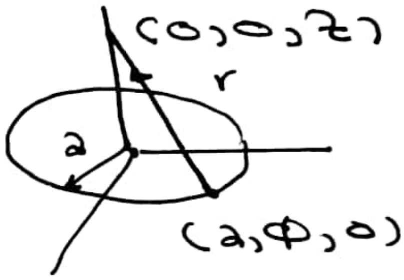
$V = \text{Constant}$

$W = 0$

$V_A = V_B$

- potential due to uniform Charged ring

$$V = \frac{Q}{4\pi\epsilon r}$$



$\because$ line Charge

$$V = \int \frac{\rho_L dl}{4\pi\epsilon r} \rightarrow (*)$$

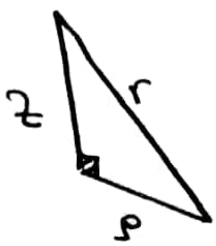
in Cylindrical Coordinates

① $dl \Rightarrow \rho d\phi$

$\rho = a$

$\therefore \boxed{dl = a\, d\phi}$

②



$$\boxed{r = \sqrt{\underbrace{\rho^2}_{a} + z^2}}$$

From 1, 2 in *

$$V = \int_0^{2\pi} \frac{\rho_L a \, d\phi}{4\pi\epsilon\sqrt{z^2+a^2}}$$

$$= \frac{\rho_L a}{4\pi\epsilon\sqrt{z^2+a^2}} \int_0^{2\pi} d\phi$$

$$= \frac{\rho_L a}{2\cancel{4}\cancel{\pi}\epsilon} \frac{\cancel{2}\cancel{\pi}}{\sqrt{z^2+a^2}}$$

$$V = \frac{\rho_L a}{2\epsilon}\left[\frac{1}{\sqrt{z^2+a^2}}\right]$$

<u>to get $\vec{E}$</u>

$$\vec{E} = -\nabla V$$

$$= -\left[\frac{dV}{d\rho}\hat{\rho} + \frac{1}{\rho}\frac{dV}{d\phi}\hat{\phi} + \frac{dV}{dz}\hat{z}\right]$$

$V \Longrightarrow$ variable with z only

$$\vec{E} = -\frac{dV}{dz}\hat{z}$$

$$\vec{E} = -\frac{d}{dz}\left[\frac{\rho_L a}{2\epsilon}(z^2+a^2)^{-1/2}\right]\hat{z}$$

$$= +\frac{\rho_L a}{2\epsilon}\cdot +\frac{1}{\cancel{2}}(z^2+a^2)^{-3/2}\cdot \cancel{2}z \quad \hat{z}$$

$$\vec{E} = \frac{\rho_L a z}{2\epsilon}\cdot\frac{1}{(z^2+a^2)^{3/2}}\hat{z}$$