Electromagnetic Fields
Course Code: ECE262
Spring Semester Exam.

Model Answer

Faculty of Engineering
Mansoura University

BME Program
Level 200
Exam Date: 13-5-2019
Allowed Time: 2 Hours

Open-Sheet Exam.

Attempt all questions. Assume any missed data. Full mark is 50.

Q.1.a) *State Coulomb's law.* Three point charges $Q_1 = 5nC$, $Q_2 = -2nC$, $Q_3 = 3nC$ are located at (1,1,1), (0,4,0), (2,0,0) respectively. Find the force on the charge Q_3. **[5 Marks]**

Q.1.b) Derive an expression for the potential of a circular *ring* of radius *'a'* located in the *x-y* plane, with its center located at the origin, at a point *(0,0,z)*. Consider the uniform linear charge density of the ring to be 'ρ_L'. Find the field at the center of the ring. **[5 Marks]**

Q.1.c) Two uniform surface charge densities of *+0.7 nC/m²* and *-0.5 nC/m²* exist on the planes *x=3m* and *x=-3m*, respectively. Find *E* at the point *P (2,4,6)*. **[5 Marks]**

Q.2.a) The spherical surfaces *r=2, 4, 6 m* contain uniform surface charge densities of *5, -10,* and *10 nC/m²*, respectively. Calculate and plot the electric field for $0 \le r \le 8 \;\; m$. **[7 Marks]**

Q.2.b) Given that $\overline{D} = 30e^{-\rho}\hat{a}_\rho - 2z\hat{a}_z$ C/m², Verify the divergence theorem for the volume enclosed by the cylinder ρ=2, $0 \le z \le 5$. **[7 Marks]**

Q.3.a) Given $\overline{E} = 5y^2\hat{a}_x + 5x^2\hat{a}_y V/m$. Determine the work involved in carrying a charge of *2μC* from *(0,-2,8)* to *(5,3,23)* along the straight line joining the two points. **[7 Marks]**

Q.3.b) Use *Laplace equation*, in spherical coordinates, to determine the capacitance between the two surfaces $\theta = const.$ with potential V_o and $\theta = \pi/2$ with potential $V = 0$. Assume the conical surface has length r_1. **[7 Marks]**

Q.4.a) Use *Biot-Savart* law to find the magnetic field due to an infinite line carrying current I, placed along the *z*-axis, at a point on the *y*-axis. **[7 Marks]**

Q.4.b) Write down Maxwell's equations for steady electric and magnetic fields in point form. If the magnetic field intensity is given as $\overline{H} = 2\rho\hat{a}_\rho + z\hat{a}_z$, find the current density *'J'*. **[7 Marks]**

My best wishes to all of you!

Assoc. Prof. Hossam El-Din Moustafa

Coloumb's law "2 marks"

- The magnitude of mutual force between two point charges is directly proportional to product of their charges and inversly propoitional to square of distance between them
- The direction is defined as unit vector joining between the two points

$$F \propto Q_1 Q_2$$

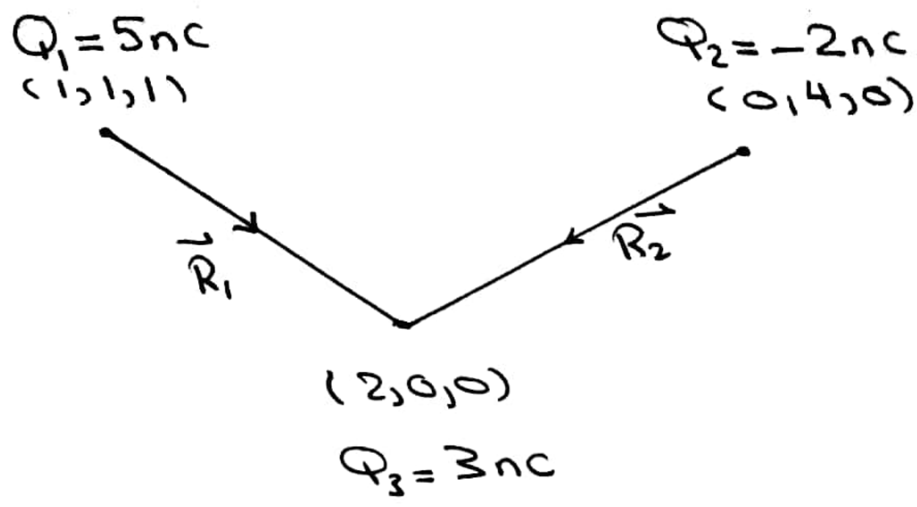
$$F \propto \frac{1}{R^2}$$

$$F \propto \frac{Q_1 Q_2}{R^2}$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi \epsilon R^2} \hat{R}$$

$$\hat{R} = \frac{\vec{R}}{|R|}$$

$$\therefore \vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon} \cdot \frac{\vec{R}}{|R|^3}$$ Newton



- $\vec{F}_T = \vec{F}_1 + \vec{F}_2$

- $\vec{F}_1 = \frac{Q_1 Q_3}{4\pi\epsilon} \cdot \frac{\vec{R}_1}{|R_1|^3}$

$$\vec{R}_1 = (2,0,0) - (1,1,1)$$

$$\vec{R}_1 = 1\hat{x} - 1\hat{y} - 1\hat{z}$$

$$|R_1| = \sqrt{1^2+1^2+1^2} \Rightarrow |R_1| = \sqrt{3}$$

$$\vec{F}_1 = \frac{5*10^{-9} * 3*10^{-9}}{4\pi(8.85*10^{-12})} \cdot \frac{1\hat{x} - 1\hat{y} - 1\hat{z}}{(\sqrt{3})^3}$$

" 1 mark "

$$\vec{F}_1 = 25.96\hat{x} - 25.96\hat{y} - 25.96\hat{z} \quad nN$$

$$\cdot \ \vec{F}_2 = \frac{Q_2 Q_3}{4\pi\epsilon} \cdot \frac{\vec{R}_2}{|R_2|^3}$$

$$\vec{R}_2 = (2,0,0) - (0,4,0)$$

$$\vec{R}_2 = 2\hat{x} - 4\hat{y}$$

$$|R_2| = \sqrt{2^2 + 4^2} \Longrightarrow |R_2| = \sqrt{20}$$

$$\vec{F}_2 = \frac{-2*10^{-9} * 3*10^{-9}}{4\pi(8.85*10^{-12})} \cdot \frac{2\hat{x} - 4\hat{y}}{(\sqrt{20})^3}$$

"1 mark"

$$\vec{F}_2 = -1.206\,\hat{x} + 2.413\,\hat{y} \quad nN$$

$$\therefore F_T = \vec{F}_1 + \vec{F}_2$$

$$= 25.96\,\hat{x} - 25.96\,\hat{y} - 25.96\,\hat{z}$$

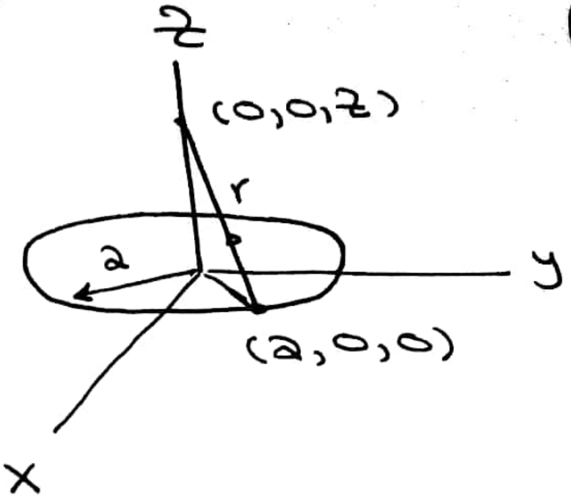
$$+ -1.206\,\hat{x} + 2.413\,\hat{y} + 0\,\hat{z}$$

$$\vec{F}_T = 24.75\,\hat{x} - 23.55\,\hat{y} - 25.96\,\hat{z} \quad nN$$

"1 mark"

10^{-9}

$\boxed{Q1}$ (b)



$$V = \frac{Q}{4\pi\epsilon r}$$

∵ Line Charge

$$V = \int \frac{\rho_L \, dl}{4\pi\epsilon r}$$

in Cylindrical system

(1) $dl \Rightarrow \rho \, d\phi$, $\rho = a$ $\stackrel{\therefore}{\Rightarrow}$ $\boxed{dl = a \, d\phi}$

(2)

$r = \sqrt{z^2 + a^2}$

$$V = \int_0^{2\pi} \frac{\rho_L a \, d\phi}{4\pi\epsilon\sqrt{z^2+a^2}}$$

$$= \frac{\rho_L a}{4\pi\epsilon\sqrt{z^2+a^2}} \int_0^{2\pi} d\phi$$

$$= \frac{\rho_L a \, 2\cancel{\pi}}{{}_2\cancel{4}\cancel{\pi}\epsilon\sqrt{z^2+a^2}}$$

$$\therefore \quad \boxed{V = \frac{\rho_L a}{2\epsilon} \cdot \left[\frac{1}{\sqrt{z^2+a^2}} \right]}$$ "2 marks"

$$\vec{E} = -\nabla V$$

$$= -\left[\frac{dv}{d\rho}\hat{\rho} + \frac{1}{\rho}\frac{dv}{d\phi}\hat{\phi} + \frac{dv}{dz}\hat{z} \right]$$

$$\because \quad V = \frac{\rho_L a}{2\epsilon} \cdot \frac{1}{\sqrt{z^2+a^2}}$$ $\Rightarrow$ vary with z only

$$\vec{E} = -\frac{dv}{dz}\hat{z}$$

$$= -\frac{d}{dz}\left[\frac{\rho_L a}{2\epsilon} (z^2+a^2)^{-1/2} \right]\hat{z}$$

$$= \cancel{-} \frac{\rho_L a}{2\epsilon} \cdot \cancel{-} \frac{1}{\cancel{2}} (z^2+a^2)^{-3/2} \cdot \cancel{2} z \,\hat{z}$$

$$\boxed{\vec{E} = \frac{\rho_L a}{2\epsilon} \cdot \frac{z}{(z^2+a^2)^{3/2}}\hat{z}}$$ "2 marks"

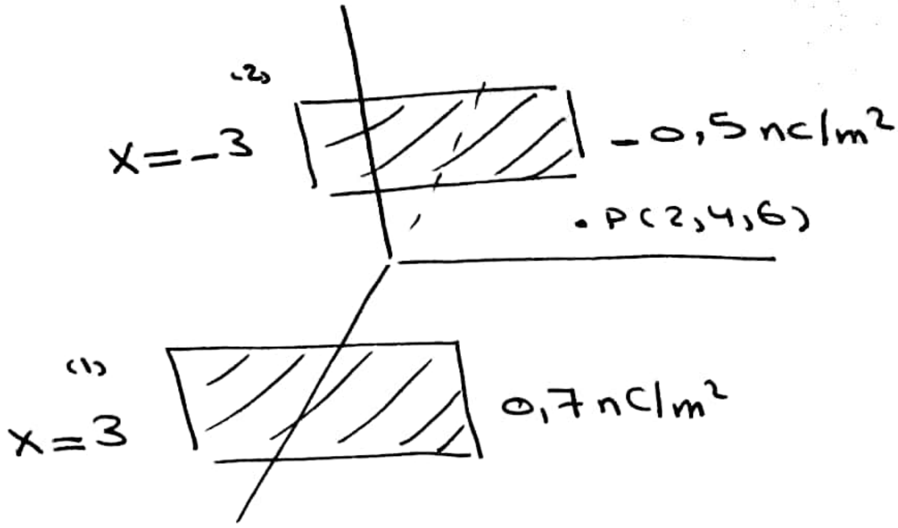
at center $\underline{z=0}$

$$\therefore \quad \boxed{\vec{E} = 0}$$ "1 mark"

Q1 (c)

$$\vec{E}_s = \frac{\rho_s}{2\epsilon}\hat{n}$$

$$\vec{E}_T = \vec{E}_{s_1} + \vec{E}_{s_2}$$



$$\cdot\ \vec{E}_{s_1} = \frac{\rho_{s_1}}{2\epsilon}(-\hat{x})$$

$$= -\frac{0{,}7 * 10^{-9}}{2 * 8.85 * 10^{-12}}\hat{x}$$

$$\boxed{\vec{E}_{s_1} = -39{,}548\,\hat{x}}$$ "2 marks"

$$\cdot\ \vec{E}_{s_2} = \frac{\rho_{s_2}}{2\epsilon}(\hat{x})$$

$$= \frac{-0{,}5 * 10^{-9}}{2 * 8.85 * 10^{-12}}\hat{x}$$

$$\boxed{\vec{E}_{s_2} = -28.248\,\hat{x}}$$ "2 marks"

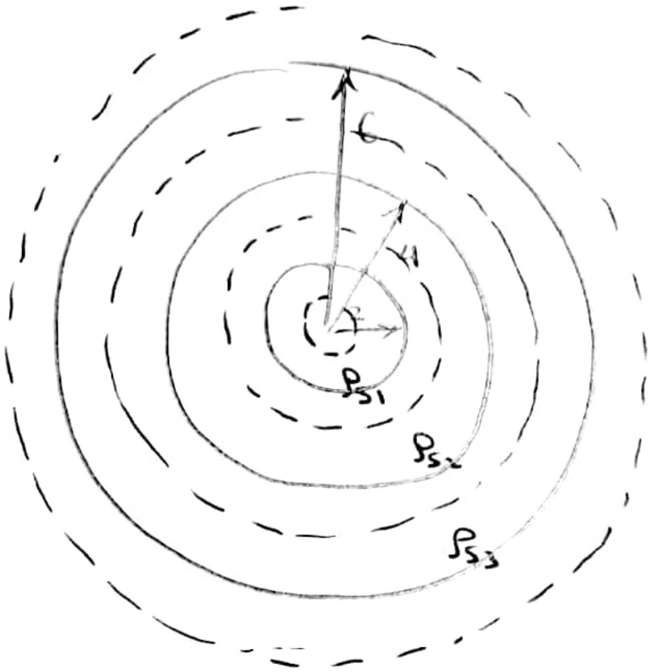
$$\vec{E}_T = -67{,}796\,\hat{x}\ \text{V/m}$$ "1 mark"

Q2 (a)

$r_1 = 2 \rightarrow \rho_{s_1} = 5\ nC/m^2$

$r_2 = 4 \rightarrow \rho_{s_2} = -10\ nC/m^2$

$r_3 = 6 \rightarrow \rho_{s_3} = 10\ nC/m^2$



<u>using Gauss law</u>

$$Q_{enc} = \oint \vec{D} \cdot d\vec{S}$$

$\because$ spherical surface

$$\therefore \underline{Q_{enc} = D_r\, 4\pi r^2}$$

$$\because D = \varepsilon E$$

$$\therefore \underline{E = \frac{D}{\varepsilon}}$$

(1) $r < 2$

$Q_{enc} = 0$

$D_r = 0$

$\boxed{E_r = 0}$

(2) $2 < r < 4$

$Q_{enc} = \rho_{s_1} S_1$

$= 5 * 4\pi (2)^2$

$20 * 4\cancel{\pi} = D_r \, 4\cancel{\pi} \, r^2$

$D_r = \frac{20}{r^2}$

$\boxed{E_r = \frac{20}{\varepsilon r^2}}$

(3) $4 < r < 6$

$Q_{enc} = \rho_{s_1} S_1 + \rho_{s_2} S_2$

$= 5 * 4\pi (2)^2 - 10 * 4\pi (4)^2$

$= -140 * 4\pi$

$-140 * 4\cancel{\pi} = D_r \, 4\cancel{\pi} \, r^2$

$$D_r = \frac{-140}{r^2}$$

$$\boxed{E_r = \frac{-140}{\varepsilon r^2}}$$

(4) $\underline{r > 6}$

$$Q_{enc} = \rho_{s_1} S_1 + \rho_{s_2} S_2 + \rho_{s_3} S_3$$

$$= 5 * 4\pi (2^2) - 10 * 4\pi (4)^2 + 10 * 4\pi (6)^2$$

$$= 220 * 4\pi$$

$$220 * \cancel{4\pi} = D_r \cancel{4\pi} r^2$$

$$D_r = \frac{220}{r^2}$$

$$\boxed{E_r = \frac{220}{\varepsilon r^2}}$$

$$E_r = \begin{cases} 0 & ; \; r < 2 \\ \dfrac{20}{\varepsilon_1 r^2} & ; \; 2 < r < 4 \\ \dfrac{-140}{\varepsilon_1 r^2} & ; \; 4 < r < 6 \\ \dfrac{220}{\varepsilon_1 r^2} & ; \; r > 6 \end{cases}$$

r	2^-	2^+	3	4^-	4^+	5	6^-	6^+	7	8
E_r	0	565	251.1	141.2	-988.7	-632.8	-439.4	690.52	507.32	388.42

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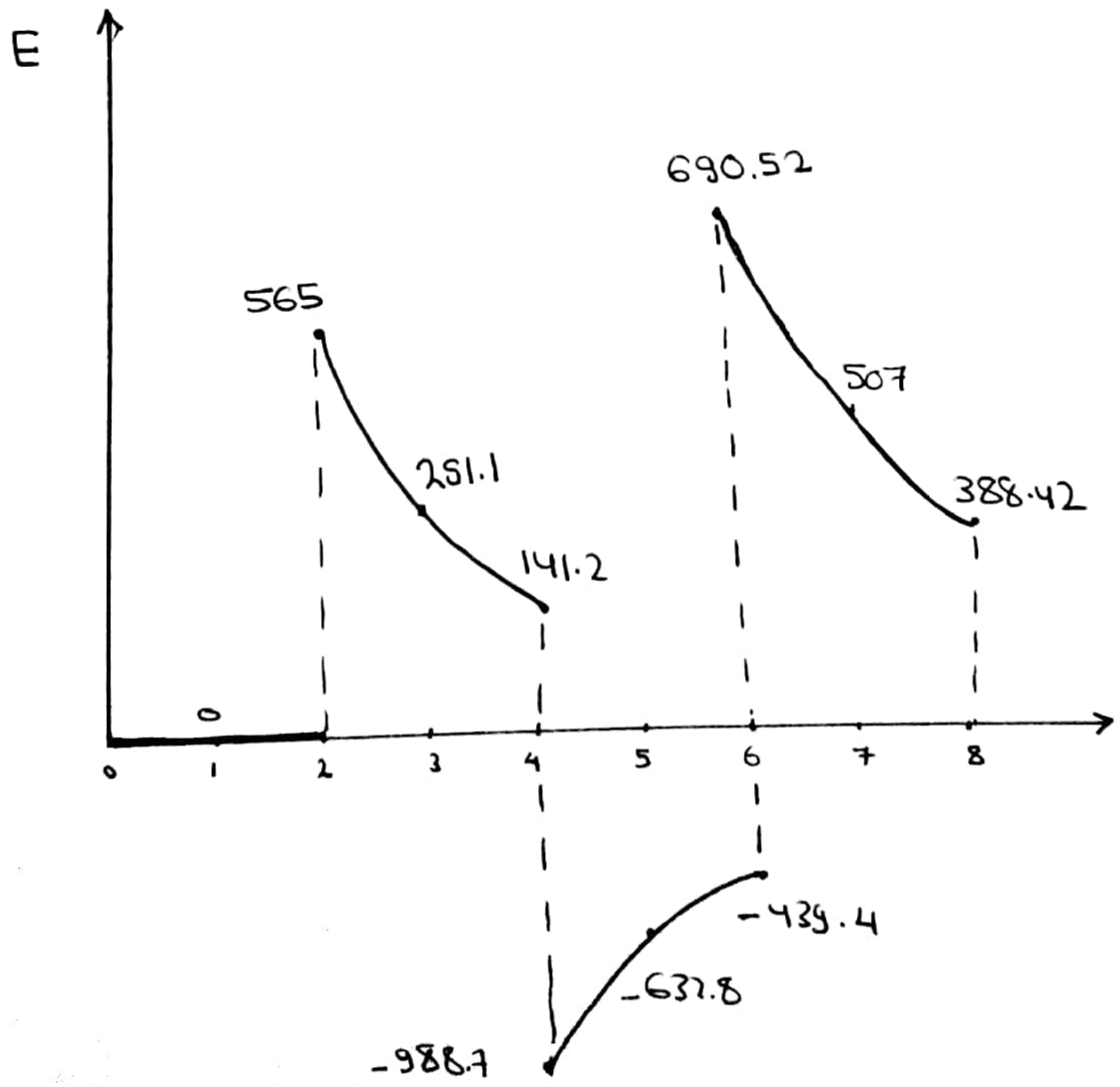


$\boxed{Q2}$ (b)

(1) using divergence

$$\nabla \cdot \vec{D} = \rho_v$$

$$= \frac{1}{\rho}\left[\frac{d}{d\rho}(\rho D_\rho) + \frac{d}{dz}(\rho D_z)\right]$$

$$= \frac{1}{\rho}\left[\frac{d}{d\rho}(\rho \cdot 30e^{-\rho}) + \frac{d}{dz}(-2z \cdot \rho)\right]$$

$$= \frac{1}{\rho}\left[(\rho \cdot -30e^{-\rho} + 30e^{-\rho}) - 2\rho\right]$$

$$\therefore \rho_v = \frac{-30\rho e^{-\rho} + 30e^{-\rho}}{\rho} - 2$$

$$Q = \iiint \rho_v \, dv$$

$$= \int_0^5 \int_0^{2\pi} \int_0^2 \left(\frac{-30\rho e^{-\rho} + 30e^{-\rho}}{\rho} - 2\right) \rho \, d\rho \, d\phi \, dz$$

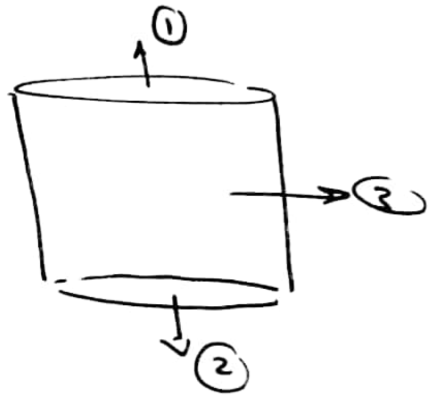
$$= \int_0^2 (-30\rho e^{-\rho} + 30e^{-\rho} - 2\rho) d\rho * \underbrace{\int_0^{2\pi} d\phi}_{2\pi} * \underbrace{\int_0^5 dz}_{5}$$

$$=\left[-30\int_0^2 \rho e^{-\rho}\, d\rho + 30\int_0^2 e^{-\rho}\, d\rho - 2\int_0^2 \rho\, d\rho\right] * 2\pi * 5$$

$$= \left[-17{,}82 + 25.94 - 4\right] * 10\pi$$

$$\therefore \boxed{Q = 41{,}2\pi = 129.43\ C} \rightarrow \text{L.H.S}$$

(2) using Gauss



$$Q = \int \vec{D} \cdot d\vec{S}$$

1. $z = 5 \rightarrow dS_1 = \rho\, d\rho\, d\phi\, \hat{z}$
2. $z = 0 \rightarrow dS_2 = -\rho\, d\rho\, d\phi\, \hat{z}$
3. $\rho = 2 \rightarrow dS_3 = \rho\, d\phi\, dz\, \hat{\rho}$

$$* \quad Q_1 = \iint (30 e^{-\rho}\hat{\rho} - 2z\hat{z})(\rho\, d\rho\, d\phi\, \hat{z})$$

$$\hat{\rho} \cdot \hat{z} = 0$$

$$\hat{z} \cdot \hat{z} = 1$$

$$= \int_0^{2\pi}\int_0^2 -2z \rho\, d\rho\, d\phi$$

$$= -2\underbrace{z}_{5}\underbrace{\int_0^2 \rho\, d\rho}_{2}\underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$$= -2(5) * 2 \quad 2\pi$$

$$\boxed{Q_1 = -40\pi C}$$

$$* \quad Q_2 = \iint (30e^{-\rho}\hat{\rho} - 2z\hat{z}) \cdot (-\rho\, d\rho\, d\phi\, \hat{z})$$

$$= \int_0^{2\pi}\int_0^{2} 2z\, \rho\, d\rho\, d\phi$$

$$= 2\underbrace{z}_{0}\int_0^2 \rho\, d\rho \int_0^{2\pi} d\phi$$

$$\boxed{Q_2 = 0}$$

$$* \quad Q_3 = \iint (30e^{-\rho}\hat{\rho} - 2z\hat{z})(\rho\, d\phi\, dz\, \hat{\rho})$$

$$\hat{\rho} \cdot \hat{\rho} = 1$$

$$\hat{z} \cdot \hat{\rho} = 0$$

$$= \int_0^5 \int_0^{2\pi} 30e^{-\rho}\rho\, d\phi\, dz$$

$$= 30 e^{-\rho}\Big|_{\rho=2} \cdot \rho \cdot \int_0^{2\pi} d\phi \cdot \int_0^{5} dz$$

$$= 30 e^{-2} * 2 * 2\pi * 5$$

$$\boxed{\varphi_3 = 81.2\pi}$$

$$\varphi_T = \varphi_1 + \varphi_2 + \varphi_3$$

$$= -40\pi + 0 + 81.2\pi$$

$$\varphi_T = 41.2\pi = 129.43C \longrightarrow R.H.S.$$

$$\therefore \quad L.H.S = R.H.S$$

$\boxed{Q3}$ (a) $\vec{E} = 5y^2\,\hat{x} + 5x^2\,\hat{y}$

$$Q = 2\mu C = 2*10^{-6}\,C$$

From $(0,-2,8)$ to $(5,3,23)$

Straight line eq

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 0}{5 - 0} = \frac{y + 2}{3 + 2} = \frac{z - 8}{23 - 8}$$

$$\frac{x}{\cancel{5}} = \frac{y+2}{\cancel{5}}$$

$$x = y + 2$$

$$dx = dy$$

$$W = -Q \int \vec{E} \cdot d\vec{l}$$

$$= -2*10^{-6}\left[\int (5y^2\,\hat{x} + 5x^2\,\hat{y})(dx\,\hat{x} + dy\,\hat{y})\right]$$

$$= -2*10^{-6}\left[\int 5y^2\,\underbrace{\cancel{dx}}_{dy} + 5x^2\,\underbrace{\cancel{dy}}_{dx}\right]$$

$$= -2*10^{-6}\left[\int_{-2}^{3} 5y^2\,dy + \int_{0}^{5} 5x^2\,dx\right]$$

$$= -2*10^{-6}\left[\frac{175}{3} + \frac{625}{3}\right]$$

$$= -2*10^{-6}\left[\frac{800}{3}\right]$$

$$\therefore \quad W = -5,33*10^{-4} \text{ joule}$$

|Q3| (b)

$\theta = \alpha$ "Constant" $\rightarrow V = V_o$

$\theta = \frac{\pi}{2} \longrightarrow V = 0$

(1) System $\longrightarrow$ spherical

(2) Variation with θ

(3) $\nabla^2 V = 0$

$$\frac{1}{r^2 \sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{dV}{d\theta} \right) = 0$$

$* \; r^2 \sin\theta$

$$\frac{d}{d\theta} \left[\sin\theta \frac{dV}{d\theta} \right] = 0$$

- integrate 1st time

$$\sin\theta \frac{dV}{d\theta} = A$$

- integrate 2nd time

$$V = \int \frac{A}{\sin\theta} d\theta$$

$$\sin\theta = 2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$$

$$V = \int \frac{A}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\, d\theta \quad * \quad \frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$V = \int \frac{A\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}}\, d\theta$$

$$= \frac{A}{2}\int \frac{\cos\theta/2}{\sin\theta/2} \cdot \frac{1}{\cos^2\theta/2}\, d\theta$$

$$A \int \frac{1/2}{\tan\theta/2} \cdot \sec^2\theta/2\, d\theta$$

$$\therefore \quad V = A\ln\left(\tan\left(\tfrac{\theta}{2}\right)\right) + B$$

Substitute with given B.C

- $V = 0,\ \theta = \frac{\pi}{2} \Rightarrow 0 = A\ln\left(\tan\left(\frac{\pi/2}{2}\right)\right) + B$ $\quad 0$

$$\therefore \underline{B = 0}$$

- $V = V_o,\ \theta = \alpha \Rightarrow V_o = A\ln\left(\tan\left(\frac{\alpha}{2}\right)\right) + \cancel{B}^{\,0}$

$$\therefore A = \frac{V_o}{\ln\left(\tan\left(\frac{\alpha}{2}\right)\right)}$$

$$\therefore \boxed{V = \frac{V_o}{\ln\left[\tan\left(\frac{\alpha}{2}\right)\right]} \ln\left(\tan\left(\frac{\theta}{2}\right)\right)}$$

(4) $\vec{E} = -\nabla V$

$$= -\frac{1}{r}\frac{dV}{d\theta}\hat{\theta}$$

$$= -\frac{1}{r}\frac{A}{\sin\theta}\hat{\theta}$$

$$= -\frac{1}{r}\frac{\frac{V_o}{\ln(\tan(\frac{\alpha}{2}))}}{\sin\theta}\hat{\theta}$$

$$\therefore \boxed{\vec{E} = -\frac{1}{r}\frac{\frac{V_o}{\ln\left[\tan\left(\frac{\alpha}{2}\right)\right]}}{\sin\theta}\hat{\theta}}$$

(5) $\vec{D} = \varepsilon\vec{E}$

$$\therefore \boxed{\vec{D} = -\frac{\varepsilon}{r}\frac{\frac{V_o}{\ln\left[\tan\frac{\alpha}{2}\right]}}{\sin\theta}\hat{\theta}}$$

(6) $\rho_s = |D|$

$$\therefore \boxed{\rho_s = \frac{\varepsilon}{r}\frac{\frac{V_o}{\ln\left[\tan\frac{\alpha}{2}\right]}}{\sin\theta}}$$

(7) $\varphi = \int \vec{D} \cdot d\vec{s}$

$$= \iint \frac{-\epsilon V_0}{r\sin\theta \ln[\tan(\frac{\alpha}{2})]} \hat{\theta} \cdot r\sin\theta \, dr \, d\phi \, \hat{\theta}$$

$\hat{\theta} \cdot \hat{\theta} = 1$

$$= \frac{-\epsilon V_0}{\ln[\tan(\frac{\alpha}{2})]} \int_0^{r_1} dr \int_0^{2\pi} d\phi$$

$$\Rightarrow \quad -\ln[\tan(\frac{\alpha}{2})] = \ln(\tan(\frac{\alpha}{2})^{-1})$$

$$= \ln[\cot(\frac{\alpha}{2})]$$

$$= \frac{\epsilon V_0}{\ln[\cot(\frac{\alpha}{2})]} \cdot r_1 \cdot 2\pi$$

$$\therefore \quad \varphi = \frac{2\pi r_1 \epsilon V_0}{\ln[\cot(\frac{\alpha}{2})]}$$

(8) $$C = \frac{Q}{\Delta V}$$

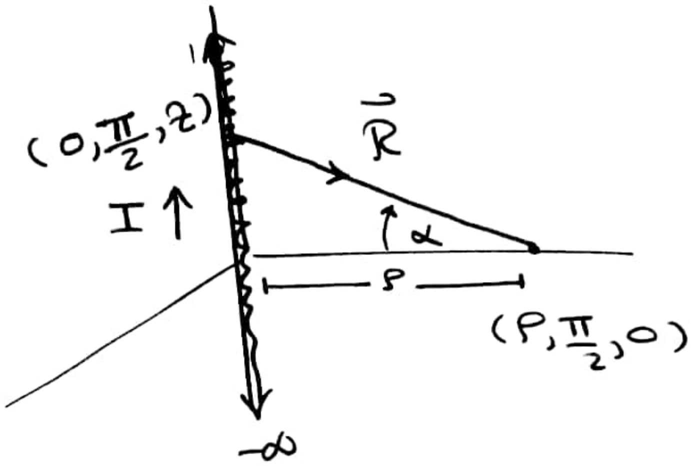
$$= \frac{\dfrac{2\pi r_1 \epsilon V_0}{\ln\left[\cot\left(\frac{\alpha}{2}\right)\right]}}{V_0 - 0}$$

$$= \frac{2\pi r_1 \epsilon \cancel{V_0}}{\cancel{V_0} \ln\left[\cot\left(\frac{\alpha}{2}\right)\right]}$$

$$\therefore \boxed{C = \frac{2\pi\epsilon r_1}{\ln\left[\cot\left(\frac{\alpha}{2}\right)\right]}}$$

Q4 (a) using Biot-Savart

$$\vec{H} = \int \frac{I\,\vec{dl}}{4\pi} \times \frac{\vec{R}}{|R|^3}$$



$$\vec{dl} = dz\,\hat{z}$$

$$\vec{R} = (\rho, \tfrac{\pi}{2}, 0) - (0, \tfrac{\pi}{2}, z)$$

$$\vec{R} = \rho\,\hat{\rho} - z\,\hat{z}$$

$$|R| = \sqrt{\rho^2 + z^2}$$

$$\vec{H} = \underbrace{\int_{-\infty}^{\infty}}_{\text{infinite line}} \frac{I\,dz\,\hat{z}}{4\pi} \times \frac{\rho\,\hat{\rho} - z\,\hat{z}}{(\sqrt{\rho^2+z^2})^3}$$

$$\begin{tikzcd} & \hat{\phi} \arrow[dr, bend left] & \\ \hat{\rho} \arrow[ur, bend left] & & \hat{z} \arrow[ll] \end{tikzcd}$$

$$\hat{z} \times \hat{\rho} = +\hat{\phi}$$

$$\hat{z} \times \hat{z} = 0$$

$$\vec{H} = \int_{-\infty}^{\infty} \frac{I \rho \, dz \; \hat{\phi}}{4\pi (\rho^2 + z^2)^{3/2}}$$

$$z = \rho \tan\alpha$$

$$z^2 = \rho^2 \tan^2\alpha$$

$$dz = \rho \sec^2\alpha \, d\alpha$$

$$z = -\infty \longrightarrow \alpha = -\frac{\pi}{2}$$

$$z = \infty \longrightarrow \alpha = \frac{\pi}{2}$$

$$\vec{H} = \frac{I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\rho^2 \sec^2\alpha \, d\alpha}{(\rho^2 + \rho^2 \tan^2\alpha)^{3/2}} \hat{\phi}$$

$$\vec{H} = \frac{I}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\rho^2 \sec^2\alpha \, d\alpha}{\left(\rho^2 (\underbrace{1 + \tan^2\alpha}_{\sec^2\alpha})\right)^{3/2}}$$

$$= \frac{I}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cancel{\rho^2} \cancel{\sec^2\alpha} \, d\alpha}{\rho^{\cancel{3}} \sec^{\cancel{3}}\alpha} \hat{\phi}$$

$$= \frac{I}{4\pi\rho} \int_{-\pi/2}^{\pi/2} \frac{d\alpha}{\sec\alpha} \hat{\phi}$$

$$= \frac{I}{4\pi\rho} \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha \, \hat{\phi}$$

$$= \frac{I}{4\pi\rho} * \sin\alpha \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \hat{\phi}$$

$$= \frac{I}{4\pi\rho} \Big[\underbrace{\sin(\tfrac{\pi}{2})}_{1} - \underbrace{\sin(-\tfrac{\pi}{2})}_{-1} \Big] \hat{\phi}$$

$$= \frac{I}{4\pi\rho} \cdot 2 \, \hat{\phi}$$

$$\therefore \vec{H} = \frac{I}{2\pi\rho} \hat{\phi}$$

Q4 (b)

Maxwell equations for steady Electric and Magnetic Field in point form

① Gauss "Electric" :

$$\nabla \cdot \vec{D} = \rho_v$$

② Electrostatic Electric Field :

$$\nabla \times \vec{E} = 0$$

③ Amperé's law :

$$\nabla \times \vec{H} = \vec{J}$$

④ Gauss "Magnetic" :

$$\nabla \cdot \vec{B} = 0$$

<u>given</u> $\vec{H} = 2\rho\,\hat{\rho} + z\,\hat{z}$

<u>find</u> $\vec{J} = ?$

<u>using Curl's eq</u> $\nabla \times \vec{H} = \vec{J}$

in Cylindrical System

$$\nabla \times \vec{H} = \frac{1}{\rho}\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \left|\begin{array}{ccc} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{d}{d\rho} & \frac{d}{d\phi} & \frac{d}{dz} \\ 2\rho & 0 * \rho & z \end{array}\right| \end{array}$$

$$= \frac{1}{\rho}\Big[+\hat{\rho}\left(\frac{d}{d\phi} z - \frac{d}{dz} 0\right)$$

$$-\rho\hat{\phi}\left(\frac{d}{d\rho} z - \frac{d}{dz} 2\rho\right)$$

$$+\hat{z}\left(\frac{d}{d\rho} 0 - \frac{d}{d\phi} 2\rho\right)$$

$\therefore \quad \underline{\vec{J} = 0}$