Almansoura University
Faculty of Engineering
Biomedical Engineering Prog.

Electromagnetic Theory
Final Exam
Time allowed: 2hours

Answer the following questions: (Assuming any missing data)

1) Point charges of 50 nC each are located at A(1,0,0), B(-1,0,0), C(0,1,0), and D(0,-1,0) in free space. Find the total force on the charge at A.
(10 degrees)

2) A uniform volume charge density of 80 $\mu C/m^3$ is present throughout the region 8 mm < r < 10 mm. **(10 degrees)**
   a- Find the total charge inside the spherical surface r=10 mm.
   b- Find D_r at r=10 mm.
   c- If there is no charge for r> 10 mm, find D_r at r=20 mm.

3) If the potential is given by $V=80\ r^{0.6}$. Assuming free space conditions, find: **(5 degrees)**
   a- The electric field E,
   b- The volume charge density at r=0.5 m, and
   c- The total charge lying within the surface r=0.6 m.

4) The surface x=0 separates two perfect dielectrics. For x>0, let $\epsilon_{r1}=3$, while $\epsilon_{r2}=5$ where x<0. If $E1=80a_x-60a_y-30a_z$ V/m. Find: **(5 degrees)**
   a- E_{t1},
   b- E_{t2},
   c- D_{N1}, and
   d- D_{N2}.

5) a- Find **H** in Cartesian components at p(2, 3, 4) if there is a current filament on the z axis carrying 8 mA in the a_z direction. **(10 degrees)**

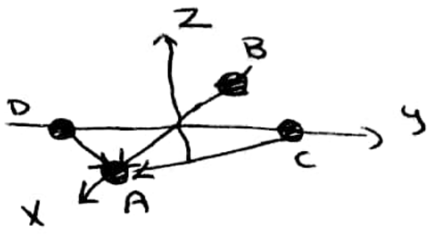
   b- Repeat if the filament is located at x = -1, y = 2.

   c- Find **H** if both filaments are present.

6) For a plane wave propagates in a lossless material, find: **(10 degrees)**
   a- Attenuation constant,
   b- Phase constant,
   c- Phase velocity,
   d- Group velocity, and
   e- Wave impedance.

$\varepsilon_r = 4$
$f = 9$ ghz

* Model answer *



1) $\vec{R}_{BA} = 2\hat{a}_x \;,\; |\vec{R}_{BA}| = 2$

$\vec{R}_{CA} = \hat{a}_x - \hat{a}_y \;,\; |\vec{R}_{CA}| = \sqrt{2}$

$\vec{R}_{DA} = \hat{a}_x + \hat{a}_y \;,\; |\vec{R}_{DA}| = \sqrt{2}$

$$\therefore \vec{F}_A = \vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} = \frac{Q^2}{4\pi\varepsilon_0}\left[\frac{\vec{R}_{BA}}{|\vec{R}_{BA}|^3} + \frac{\vec{R}_{CA}}{|\vec{R}_{CA}|^3} + \frac{\vec{R}_{DA}}{|\vec{R}_{DA}|^3}\right]$$

$$\therefore \vec{F}_A = \frac{(50\times10^{-9})^2}{4\pi\varepsilon_0}\left[\frac{2\hat{a}_x}{2^3} + \frac{\hat{a}_x - \hat{a}_y}{2^{3/2}} + \frac{\hat{a}_x + \hat{a}_y}{2^{3/2}}\right]$$

$$\boxed{\therefore \vec{F}_A = 21.505\ \hat{a}_x\ \mu N \ \#}$$

2) (a) $Q_{tot.} = \int \overset{\text{constant}}{\rho_v}\, dV = \rho_v \cdot \text{Volume}$

$= (80\times10^{-6}) \cdot \frac{4}{3}\pi\left[(10\times10^{-3})^3 - (8\times10^{-3})^3\right]$

$$\boxed{\therefore Q_{tot.} = 0.164\ nC \ \#}$$

(b) Assuming Gaussian surface as a sphere with radius $8 < r < 10$ mm

$Q_{enc.} = \oint \vec{D}\cdot d\vec{s} \Rightarrow D_r \cdot 4\pi r^2 = Q_{enc.}$

$$\therefore D_r = \frac{(80\times10^{-6})\cdot\frac{4}{3}\pi\left[r^3 - 0.008^3\right]}{4\pi r^2}$$

$$\boxed{\therefore D_r\Big|_{r=10mm} = 0.13\ \mu C/m^2}$$

(c) Assuming Gaussian surface as a sphere with radius $r > 10mm$

$\therefore Q_{enc.} = D_r \cdot 4\pi r^2$

$$\therefore D_r = \frac{0.164\times10^{-9}}{4\pi r^2} \qquad \boxed{\therefore D_r\Big|_{r=20mm} = 32.63\ nC/m^2}$$

3) (a) $\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{a}_r + 0 + 0 = -\frac{\partial}{\partial r}(80 r^{0.6})$

$\therefore \vec{E} = -48\, r^{-0.4}\, \hat{a}_r$ V/m # $\quad \therefore \vec{D} = \varepsilon\vec{E} = -48\varepsilon_0 r^{-0.4}\hat{a}_r$ C/m^2

$$\therefore \rho_v = \text{Div}\,\vec{D} = \frac{1}{r^2\cancel{\sin\theta}}\left[\frac{\partial}{\partial r}(r^2\cancel{\sin\theta}\, D_r) + 0 + 0\right]$$

b. $\therefore \rho_v = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(-48\varepsilon_0 r^{1.6}\right)\right] = \frac{-48\varepsilon_0}{r^2}\left[1.6\, r^{0.6}\right]$

$$= -76.8\,\varepsilon_0\, r^{-1.4}$$

$$\boxed{\therefore \rho_v\Big|_{r=0.5m} = -1.79\ nC/m^3 \ \#}$$

c. $Q = \int \rho_v\, dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{0.6} -76.8\,\varepsilon_0\, r^{-1.4}\, r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$= -76.8\,\varepsilon_0 \cdot 2\pi \cdot \frac{r^{1.6}}{1.6}\Big|_0^{0.6} \cdot \cos\theta\Big|_{\pi}^{0}$$

$$= -76.8\,\varepsilon_0\,(2\pi)\,\frac{0.442}{1.6}\cdot(2)$$

$$\boxed{\therefore Q_{enc.} = -2.359\ nC \ \#}$$

4) tangent $\Rightarrow \hat{a}_z, \hat{a}_y$
normal $\Rightarrow \hat{a}_x$

$\varepsilon_{r1} = 3$
$\vec{E}_1 = 80\hat{a}_x + 60\hat{a}_y - 30\hat{a}_z\ V/m$
$x = 0$
$\varepsilon_{r2} = 5$

$\therefore \vec{D}_1 = \varepsilon_1 \vec{E}_1 = 3\varepsilon_0\left[80\hat{a}_x + 60\hat{a}_y - 30\hat{a}_z\right]$

According to B.C's

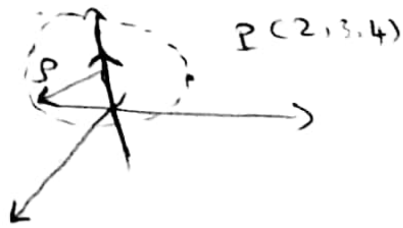
$\therefore E_{1t} = E_{2t}$ & $\therefore D_{N1} = D_{N2}$

$\therefore E_{2y} = E_{1y} = 60$
$E_{2z} = E_{1z} = -30$

$\therefore D_{1x} = D_{2x} = 240\,\varepsilon_0$

$$\boxed{\begin{array}{l} \therefore E_{t1} = 60\hat{a}_y - 30\hat{a}_z \ \ \& \ \ E_{t2} = 60\hat{a}_y - 30\hat{a}_z \\ \& \ \ D_{N1} = 240\,\varepsilon_0\,\hat{a}_x \ \ \& \ \ D_{N2} = 240\,\varepsilon_0\,\hat{a}_x \end{array}}$$

5) (a) using Ampere's law



Assume Amperian path caring with radius ρ

$\therefore \oint \vec{H}\cdot d\vec{l} = I_{enc.}$

$\therefore H_\phi \cdot 2\pi\rho = I_{enc.}$

$\therefore \vec{H} = \frac{I_{enc}}{2\pi\rho}\,\hat{a}_\phi$, $\phi = \tan^{-1}\left(\frac{3}{2}\right) = 56.31$

$$\vec{H}\Big|_{at\ P(2,3,4)} = \frac{8}{2\pi\sqrt{4+9}}\left(-\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y\right)$$

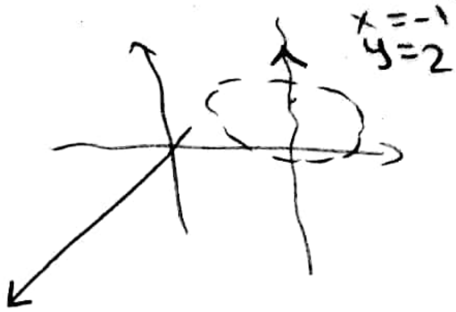
$$\boxed{\therefore \vec{H} = -0.294\,\hat{a}_x + 0.195\,\hat{a}_y \ \#\ mA/m}$$

(2)

(b) First using axis transformation

$X = x + 1$

& $Y = y - 2$



∴ observation pt will be $(3, 1, 4)$ ⟶ $\phi = \tan^{-1}\left(\frac{1}{3}\right) = 18.43$

using Ampere's law again

∴ $\vec{H} = \frac{I_{enc.}}{2\pi\rho}\,\hat{a}_\phi = \frac{8}{2\pi\sqrt{9+1}}\,(-\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y)$

$\boxed{\vec{H} = -0.127\,\hat{a}_x + 0.381\,\hat{a}_y \quad mA/m}$ #

(c) ∴ $\boxed{\vec{H_T} = \vec{H_a} + \vec{H_b} = 0.578\,\hat{a}_y - 0.421 \quad mA/m}$ #

6) (a) $\boxed{\alpha = 0 \quad Np/m}$ " because its a lossless "

(b) $B = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu_o\mu_r\,\varepsilon_o\varepsilon_r}$

$= 2\pi(9\times10^9)\sqrt{4\,\mu_o\varepsilon_o} = \boxed{120\pi \quad rad/m}$

(c) $v_{ph} = \frac{\omega}{B} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{3\times10^8}{\sqrt{4}} = \boxed{15\times10^7 \quad m/sec.}$

(d) $\boxed{V_{group} = \frac{\partial\omega}{\partial B} \quad m/s}$

(e) $\eta = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1}{4}} = \boxed{60\pi \quad \Omega}$