Electromagnetic Fields
Course Code: ECE262
Spring Semester
Mid-term exam

BME Program
Level 200
Exam Date: 31-3-2018
Allowed Time: 1 Hour

Open-Sheet Exam.

Answer four questions only. Assume any missed data. Full mark is 20.

Q.1) *State Coulomb's law.* Derive an expression for the electric field of a circular *disk* of radius *'a'* located in the *x-y* plane, with its center located at the origin, at a point *(0,0,z)*. Consider the uniform surface charge density of the disk to be 'ρ_L'. **[5 Marks]**

Q.2) A line charge of $\rho_L = 3nC/m$ along the x-axis and a uniform surface charge density of *0.2 nC/m²* exists on the plane *y = 3 m*. Find $\overline{E}$ at the point P (1,1,1). **[5 Marks]**

Q.3) A coaxial cable with inner conductor of radius *a=0.1* cm and outer conductor of radius *b=0.3* cm and linear charge densities of *0.2 nC/m* and *-0.2 nC/m* respectively. Find and sketch the electric field everywhere. **[5 Marks]**

Q.4) Given that $\overline{D} = 4r^2 a_r$ C/m², find the volume charge density ρ_V. Verify divergence theorem for the volume enclosed by this sphere. **[5 Marks]**

Q.5) For an electric dipole with charges Q and –Q spaced by distance '*d*' on the z-axis, find the potential at point '*P*' located in the y-z plane. Hence, find the electric field at point '*P*' and the equation of the equi-potential surface. **[5 Marks]**

My best wishes to all of you!

Assoc. Prof. Hossam El-Din Moustafa

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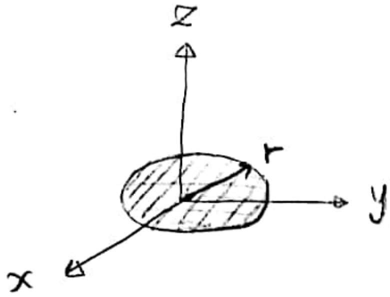
Question 1

~~Assume~~

- Estate coloumb's Law
- Derive an Expression for the electric field of a circular disk of radius $r=a$ on the x-y plane, center at origin consider uniform charge denisty



Solution

$\because$ Coloumb's law's : $E = \frac{Q}{4\pi E} \frac{\vec{R}}{|R|^3}$

$E = \frac{F}{Q}$

$\because dE = \frac{dQ}{4\pi E} \frac{\vec{R}}{|R|^3} = \frac{dQ}{4\pi E r^2} \hat{r}$

$\because dQ = \rho_s dS \qquad \therefore dQ = \rho_s \, dS$

$\therefore dE = \frac{\rho_s \, dS}{4\pi E a^2} \hat{a} = \frac{\rho_s \cancel{2\pi} a}{{}_2 4 \cancel{\pi} E a^2} \hat{a}$

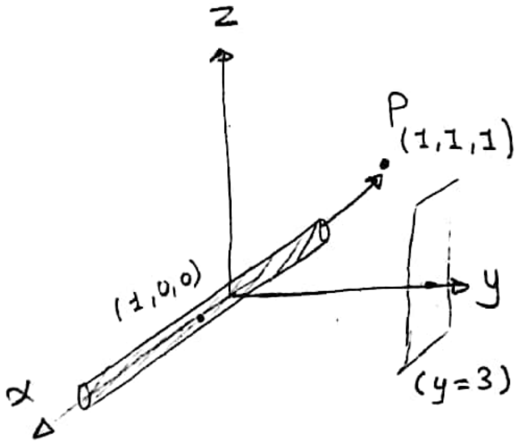
$\therefore E = \frac{\rho_s}{2E} \hat{z} = \frac{\rho_L}{2E} \hat{z} \longrightarrow \#$

Question 2

A line charge $\rho_L = 3nC/m$ along x axis

A uniform surface charge $\rho_s = 0.2\, nC/m^2$ on plane $y=3m$

Find $\overline{E}$ at Point $(1,1,1)$



Solution

$$\because \overline{E} = \overline{E}_{line} + \overline{E}_{surface}$$

$$\because \overline{E}_{surface} = \frac{\rho_s}{2\epsilon}\hat{n} = \frac{0.2\times10^{-9}}{2\times8.85\times10^{-12}} -\hat{y} = -11.3\hat{y}$$

$$\because \overline{E}_{line} = \frac{\rho_L}{2\pi\epsilon}\frac{\vec{R}}{|R|^2} \qquad \because \vec{R} = \hat{y}+\hat{z} \qquad |R| = \sqrt{2}$$

$$\therefore \overline{E}_{line} = \frac{3\times10^{-9}}{2\pi\times8.85\times10^{-12}\times 2}(\hat{y}+\hat{z}) = 27\hat{y} + 27\hat{z}$$

$$\therefore \overline{E} = 15.7\hat{y} + 27\hat{z} \longrightarrow \#$$

$$\boxed{\overline{E} = 15.7\hat{y} + 27\hat{z}}$$

Question 3

A Coaxial cable inner radius $= 0.1$ cm $\rho_L = 0.2\, nC/m$
outer radius $= 0.3$ cm $\rho_L = -0.2\, nC/m$

Find and sketch electric field every where.

Solution

First Case if the radius of Gauss surface $r<a$;

$\because Q_{inside} = 0 \qquad \therefore E_{in} = 0$

Second case if the radius of Gauss surface $a<r<b$

$$\because Q = \overline{D}S = \overline{D}\times 4\pi r^2$$

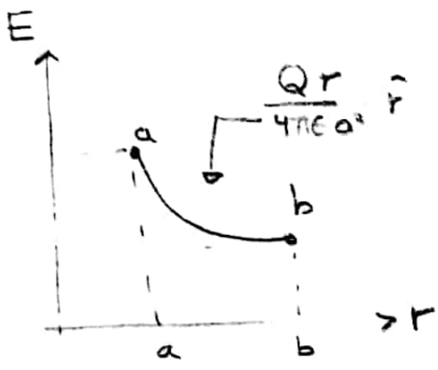
$$\because Q = \rho_v V \qquad \because \rho_v = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$\therefore Q = \frac{Q}{\frac{4}{3}\pi a^3}\frac{4}{3}\pi r^3$$

$$\therefore \overline{D} = \frac{Qr^3}{4\pi a^3 r^2} \qquad \therefore \overline{D} = \frac{Qr}{4\pi a^3} \qquad \because \overline{E} = \frac{\overline{D}}{\epsilon}$$

$$\therefore \overline{E} = \frac{Qr}{4\pi\epsilon a^3}\hat{r}$$

$$\boxed{\overline{E} = \frac{Qr}{4\pi\epsilon a^3}\hat{r}}$$



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Question 4.

Given that $\overline{D} = 4r^2\ \hat{r}\ C/m^2$

- Find the volume charge density ρ_v.
- Verify Divergence theorem for enclosed theorem.

Solution

$\because \rho_v = \nabla \overline{D} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial [h_2 h_3 D_{u_1}]}{\partial u_1}\right]$

$\therefore \rho_v = \frac{1}{r^2 \sin\theta}\left[\frac{\partial r^2 \sin\theta\, D_r}{\partial r}\right] = \frac{\partial D_r}{\partial r}$

$\therefore \rho_v = \frac{\partial 4r^2}{\partial r}\hat{r} = 8r\ \hat{r} \longrightarrow a.$

$\boxed{\rho_v = 8r\,\hat{r}}$

* Verifing Divergence theorem :

$\because \Psi = Q_{enc}$ $\quad \therefore Q_{enc} = \int_v \rho_v\, dV$

$\therefore Q_{enc} = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} 8r^3 \sin\theta\, dr\, d\theta\, d\phi = 2 \times r^4\Big|_0^{r} \times -\cos\theta\Big|_0^{\pi} \times 2\pi$

$\therefore Q_{enc} = 4\pi \times 2 \times r^4$ $\quad \therefore Q_{enc} = 8\pi r^4$

$\because Q_{enc} = \oint_s \overline{D}\, ds = \int_0^{2\pi}\int_0^{\pi} 4r^2 \times r^2 \sin\theta\, d\theta\, d\phi = 4r \times -\cos\theta\Big|_0^{\pi} \times 2\pi$

$Q_{enc} = 8\pi r^4$

$\because \oiint \overline{D}\, d\overline{S} = \iiint \rho_v\, dv$

$\therefore$ The Divergence Theorem is verified.

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