Mid Term Exam

Heat and Mass transfer

Nov. 2017

Time allowed 1 hr. Total marks 20

Question One:

a. Starting with energy balance on a rectangular volume element derive the one-dimensional steady state heat conduction equation for a plane wall with heat generation.

b. The wall of an air-condition theatre in Toronto is made up of an out layer of *30 cm* common brick *(k = 0.7 W/m.K)*, *10 cm* of glass wool insulation (*k = 0.04 W/m.K*) and an inner layer of *6 mm* wooden paneling (*k = 0.11 W/m.K*). The outer surface of the wall is at *–20°C* and the inner surface of the wooden panel is at *20°C*. Determine the heat flow rate per m^2 of the wall. If the inner measurement of the theatre is *(100×50×5)* m^3, estimate the total heat loss rate through the four walls. Find also the minimum temperature of the glass wool insulation. Neglect the edge and corner losses.

Question Two:

Engine oil at 80°C flows over a 6 m long flat plate whose temperature is 30°C with a velocity of 3 m/s. Determine the total drag force and the rate of heat transfer over the entire plate per unit width.

The properties of engine oil at the film temperature of $(T_s + T_\infty)/2$ = (80+30)/2 =55°C = 328 K are

$$\rho = 867\ \text{kg/m}^3 \qquad \upsilon = 123\times10^{-6}\ \text{m}^2/\text{s}$$
$$k = 0.141\ \text{W/m.°C} \qquad \text{Pr} = 1505$$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $$\text{Nu} = \frac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} \qquad \text{Re}_L < 5\times10^5$$

Turbulent:
$$\text{Nu} = \frac{hL}{k} = 0.037\ \text{Re}_L^{0.8}\ \text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5\times10^5 \le \text{Re}_L \le 10^7 \end{array}$$

Good Luck

Prob.1

$t_{s1} = 20°C$, $l_1 = 0.006\ m$, $k_1 = 0.11\ W/m.K$

$l_2 = 0.1\ m$, $k_2 = 0.04\ W/m.K$

$l_3 = 0.3\ m$, $k_3 = 0.7\ W/m.K$

$t_{s4} = -20\ °C$

for $1\ m^2$ of wall, $A = 1\ m^2$

the thermal resistances for the three layers are

$R_1 = l_1 / (k_1A) \quad = 0.006 / 0.11 = 0.0545\ K/W$

$R_2 = l_2 / (k_2A) \quad = 0.1 / 0.04 = 2.5\ K/W$

$R_3 = l_3 / (k_3A) \quad = 0.30 / 0.70 = 0.429\ K/W$

Total thermal resistance

$R_T = R_1 + R_2 + R_3$

$= 0.0545 + 2.5 + 0.429 = 2.984\ K/W$

Then,

$q = (t_{s1}-t_{s4}) / R_T$

$= (20 - (-20)) / 2.984$

$= 40 / 2.984 = 13.4\ W$ per m^2 of wall.

Total heat transfer area,

$A = 2(100\times5) + 2(50\times5) = 1500\ m^2$

Total heat loss rate $= 13.40\times1500$

$= 20100\ W = 20.1\ kW$

Under steady state conditions the flow of heat through each layer is constant. Therefore we can write

$q = (t_3 - t_{s4}) / R_3$

$13.4 = (t_3 - (-20)) / 0.429$

$t_3 = -14.25\ °C$

Model Answer

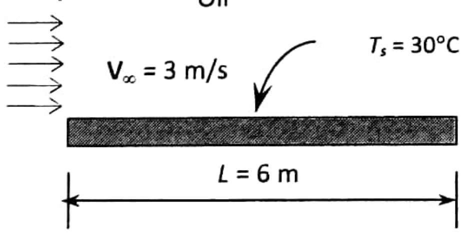
Prob 2: Hot engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is Re_{cr} = 5×10^5. **3** Radiation effects are negligible.

Properties The properties of engine oil at the film temperature of $(T_s + T_\infty)/2$ = (80+30)/2 =55°C = 328 K are (Table A-13)

$$\rho = 867 \text{ kg/m}^3 \qquad \upsilon = 123\times10^{-6} \text{ m}^2\text{/s}$$
$$k = 0.141 \text{ W/m.°C} \qquad \text{Pr} = 1505$$

Analysis Noting that L = 6 m, the Reynolds number at the end of the plate is



$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{(3 \text{ m/s})(6 \text{ m})}{123\times10^{-6} \text{ m}^2/\text{s}} = 1.46\times10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate. The average friction coefficient and the drag force per unit width are determined from

$$C_f = 1.328\,\text{Re}_L^{-0.5} = 1.328(1.46\times10^5)^{-0.5} = 0.00347$$

$$F_D = C_f A_s \frac{\rho V_\infty^2}{2} = (0.00347)(6\times1\text{ m}^2)\frac{(867 \text{ kg/m}^3)(3 \text{ m/s})^2}{2} = 81.3 \text{ N}$$

Similarly, the average Nusselt number and the heat transfer coefficient are determined using the laminar flow relations for a flat plate,

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664(1.46\times10^5)^{0.5}(1505)^{1/3} = 2908$$

$$h = \frac{k}{L}Nu = \frac{0.141 \text{ W/m.°C}}{6 \text{ m}}(2908) = 68.3 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer is then determined from Newton's law of cooling to be

$$\dot{Q} = hA_s(T_\infty - T_s) = (68.3 \text{ W/m}^2\text{.°C})(6\times1\text{ m}^2)(80-30)\text{°C} = 2.05\times10^4 \text{ W} = \mathbf{20.5\,kW}$$