Mansoura University
Faculty of Engineering

Fall Semester Exam.
Level 200
Sunday 24/1/2017

Biomedical Engineering Programs

Prof. Dr. Magdi S. El-Azab | Numerical Analysis (MTH 201) | Time allowed: 2 Hours.

Answer the following questions (Full mark 50 marks). (All computations have to be executed to four decimal places)

Question 1 [24 marks]

(a) [8 marks] Given the table of readings:

	0.25	0.5	0.75	1	1.25
	1.14	1.82	2.03	2.00	1.87

(i) Fit the curve $y = \dfrac{x}{a + bx^2}$, with the computations of $L_2 - norm$ of *the* error, to the above readings.

(ii) By the use of two different interpolation formulas approximate $y(0.4)$ and $y(0.8)$.

(b) [8 marks] Given the initial boundary-value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 2, \qquad t > 0$$

$$u(0,t) = 20, \quad u(0.5,t) = 0, \qquad t > 0$$

$$u(x,0) = 20\cos\left(\frac{\pi x}{4}\right), \qquad 0 < x < 2, \qquad t > 0$$

Taking $h = 0.4$, approximate $u(x, 0.16)$, $0 \le x \le 0.5$ correct to three decimal places.

(c) [8 marks] Derive Newton-Raphson formula to solve the equation $f(x) = 0$. Hence use this method to find Solve numerically for x the equation

$$x^4 - 6.255x^3 + 10.78x^2 - 6.255x + 9.78 = 0,$$

correct to five decimal places.

باقى الاسئلة فى الصفحة التالية

Question 2 [27 marks]

(a) [9 marks] Given the initial value problem:

$$y' + \frac{2x + y^2}{xy} = 0, \quad y(1.2) = 1,$$

(i) Evaluate $y(1.4)$ by Runge-Kutta approximation of order 4,

(ii) Find $y(1)$ by the use of Taylor method,

(iii) Find $y(1.6)$ by the use of Euler method.

(iv) Use a predictor-corrector method to evaluate $y(1.8)$.

(b) [9 marks] Starting with the initial values

$$\left(x_1^{(0)},\ x_2^{(0)},\ x_3^{(0)},\ x_4^{(0)}\right) = (2,\ 2,\ 5,\ 5),$$

solve the following system of equations correct to three decimal places

$$\begin{array}{rcrcrcrcr} 10x_1 & + & x_2 & - & 2x_3 & + & x_4 & = & 10 \\ x_1 & - & 10x_2 & - & x_3 & + & x_4 & = & -20 \\ 2x_1 & - & 2x_2 & + & 10x_3 & - & 4x_4 & = & 50 \\ x_1 & - & x_2 & + & x_3 & - & 10x_4 & = & -40 \end{array}$$

(c) [9 marks] Apply the finite difference method and two iterations of Gauss-Seidel method to solve Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 12(x + y),$$

at the interior points of the square $0 \le x \le 2,\ 0 \le y \le 1.5$. Take the mesh sizes $h = k = 0.5$ and the unknown u is given to be $(4x^2 + 2y)$ on the boundaries.

With all best wishes
Dr Magdi S. El-Azab

Q:- 1) $y = \frac{x}{a + bx^2}$

$\frac{1}{y} = \frac{a}{x} + \frac{bx^2}{x}$

$\frac{1}{y} = a\frac{1}{x} + bx \quad \div x$

$\frac{1}{yx} = a\frac{1}{x^2} + b$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$Y \quad A \quad X \quad B$

Y	X	X^2	XY
3.50877	16	256	56.14032
1.09890	4	16	4.3956
0.656814	$\frac{16}{9}$	$\frac{361}{36}$	1.167669333
0.5	1	1	0.5
0.427807	0.64	0.4096	0.27379648
6.192294965	23.41778	276.5700938	62.47744222

$A\Sigma x^2 + B\Sigma x = \Sigma xy$

$A\Sigma x + Bn = \Sigma y$

$A = 0.200582045$

$B = 0.299021837$

$y = \frac{x}{(0.20) + (0.299)x^2}$

y	ε^2
1.143183767	$1.013637231 * 10^{-5}$
1.819836215	$2.682552623 * 10^{-8}$
2.037005602	$4.90784\,5938 * 10^{-5}$
2.004008016	$1.606419226 * 10^{-5}$
1.8735363	$1.250541603 * 10^{-5}$
ε	$8.781126551 * 10^{-5}$

$$\text{error} = \sqrt{\varepsilon^2} = 9.370766538 * 10^{-3}$$

(ii)

0.25	1.14				
		0.68			
0.5	1.82		-0.47		
		0.21		0.23	
0.75	2.03		-0.24		-0.09
		-0.03		0.14	
1	2		-0.1		
		-0.13			
1.25	1.87				

$X = 0.4$

$X_0 = 0.5$

$h = 0.25$

$$S = \frac{X - X_0}{h} = -0.4$$

$$y(0.4) = 1.82 + (0.21 * -0.4) + \frac{\frac{217}{125}}{2!} * -0.4(-0.4-1) + \frac{\frac{-0.24}{2!}}{2!} * -0.4(-0.4-1)$$

$$+ \frac{\frac{1043}{625}}{3!} * -0.4 * (-0.4-1) * (-0.4-2) + \frac{0.14}{3!} * -0.4 * (-0.4-1) * (-0.4-2) = 1.63744$$

la garange :-

$$f(0.8) = \frac{(X-0.5)(X-0.75)(X-1)(X-1.25)}{(0.25-0.5)(0.25-0.75)(0.25-1)(0.25-1.25)} * 1.14$$

$$+ \frac{(X-0.25)(X-0.75)(X-1)(X-1.25)}{(0.5-0.25)(0.5-0.75)(0.5-1)(0.5-1.25)} * 1.82$$

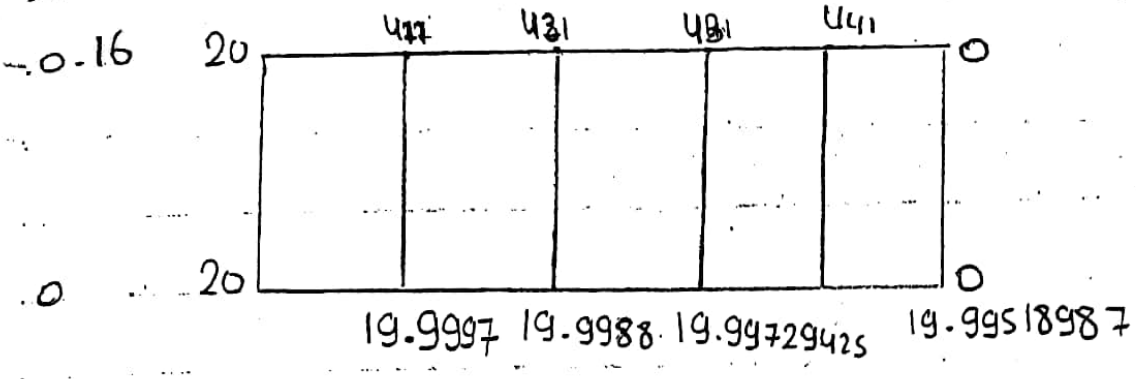
$$+ \frac{(X-0.25)(X-0.5)(X-1)(X-1.25)}{(0.75-0.25)(0.75-0.5)(0.75-1)(0.75-1.25)} * 2.03$$

$$+ \frac{(X-0.25)(X-0.5)(X-0.75)(X-1.25)}{(1-0.25)(1-0.5)(1-0.75)(1-1.25)} * 2$$

$$+ \frac{(X-0.25)(X-0.5)(X-0.75)(X-1)}{(1.25-0.25)(1.25-0.5)(1.25-0.75)(1.25-1)} * 1.87$$

$$= 2.008912$$

(b)



$$\lambda = \frac{1 * 0.16}{(0.4)^2}$$

$u_{11} = (20 + 19.9988) + (1-2) * 19.9997$ $u = \lambda(A+C) + (1-2\lambda)B$

$= 19.9991$

$u_{21} = (19.9997 + 19.9973) + (1-2) * 19.9988 = 19.9982$

$u_{31} = (19.9988 + 19.9952) + (1-2) * 19.99729425 = 19.9967$

$u_{41} = (19.9973 + 0) + (1-2) * 19.9952 = 2.1 * 10^{-3}$

(c) : $x^4 - 6.255x^3 + 10.78x^2 = 6.255x + 9.78$

$f'(x) = 4x^3 - 18.765x^2 + 21.56x - 6.255$

$$x_{n+1} = x_n + \frac{f(x)}{f'(x)}$$

0

1.563549161

4-254136071

3.804185905

3.514789529

3.340394984

3.242467077

3.191927785

3.169747507

3.163513873

3.162948316

root is **3.162943618**

Question 2:-

	$h=-0.2$	$h=0.2$	$h=0.2$	$h=0.2$	
	$X=1$	$X=1.2$	$X=1.4$	$X=1.6$	$X=1.8$
a) ① Taylor	$y_1=$ ✓	$y=1$	$y_2=$ ✓	$y=$	$y=$

$$y' = \frac{-2x+y^2}{xy} = -(2x+y^2) * (xy)^{-1} = -\frac{7}{6}$$

$$y'' = -(2+2yy') * -1 \cdot (xy)^{-2} (xy'+y) = \frac{5}{54}$$

$$y_1 = 1 + \frac{-0.2}{1!} * -\frac{7}{6} + \frac{0.2^2}{2!} * \frac{5}{54} = 1.235185185$$

② Runge-Kutta:-

$K_1 = \frac{-17}{30} = -0.567$

$K_2 = -0.668$

$K_3 = -0.703$ $\quad y_2 = 0.21683$

$K_4 = -1.396$

③ euler

$$y_3 = 0.21683 + 0.2 * \left(-\frac{2*(1.4)+(0.21683)^2}{1.4*0.21683}\right)$$

Molten		1	-5	19	9
Adam's	-9	37	-59	55	
X	1	1.2	1.4	1.6	1.8
Y	1.235	1	0.21683	-1.6589	3.321
F	-2.854	-2.83	-9.38	2.24	=2.45

Adam's $y = (-1.6589) + \frac{0.2}{24}(-9*f + 37*f - 59*f + \ldots$

$= 3.3210667$

molten $= -1.6589 + \frac{0.2}{24}(1*f + -5*f + 19*f + 9*f)$

$= -1.12073$

b)

$$x_1 = \frac{1}{10}(10 - x_2 + 2x_3 - x_4)$$

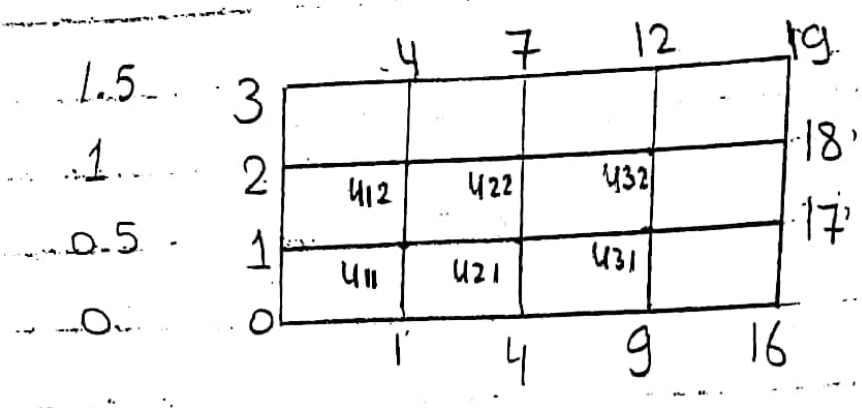
$$x_2 = \frac{1}{-10}(-20 - x_1 + x_3 - x_4)$$

$$x_3 = \frac{1}{10}(50 - 2x_1 + 2x_2 + 4x_4)$$

$$x_4 = \frac{1}{-10}(-40 - x_1 + x_2 - x_3)$$

x_1	x_2	x_3	x_4
2	2	5	5
1.3	2.13	7.166	4.6336
1.75684	1.9224444	6.8865608	4.672095 68
1.7178 58192	1.950339307	6.915334495	4.668285338
1.721204434	1.947415528	6.912555635 4	4.668634526
1.720906265	1.947698444	6.912812246	4.668602007
1.720932404	1.947672216	6.912788765	4.668604895

(c)



	4		7		12	
2	u_{12}	f	u_{22}	f	u_{32}	f
	~~20~~	18	~~30~~	24	~~30~~	30
	9.6875		13.71875		13.0664625	
1	u_{11}	f	u_{21}	f	u_{31}	f
	~~10~~	12	~~10~~	18	~~20~~	24
	7.25		14.1875		16.046875	
	1		4		9	