Exam Guidelines: This Exam contains **2** questions in **1** page.

Question (1):[30 marks]

(a)[10 marks] fit the readings

x	1	1.5	2	2.5	3	3.5	4
y	0.12	0.52	1.47	3.24	6.21	10.78	17.35

For the exponential curve $y = m\,x^c$

(b) [10 marks] Use lagrange's interpolation formula to find $f(2)$ given that

x	0	1	3	6
y	2	4	10	18

(c) [10 marks] use simple iteration method to obtain the smallest positive root of the equation $x^3 - 8x + 5 = 0$.

Question (2): [20 marks]

(a) [10 marks] use Gauss-Seidel method to solve the following system of linear equations: (10 iterations are required)

$$5x_1 - x_2 + 3x_3 = -2$$
$$x_1 + 5x_2 - 2x_3 = 10$$
$$2x_1 - 4x_2 + 10x_3 = 6$$

(b) [10 marks] obtain the value of y at $x = 0.8$ by fourth order Rung-Kutta method if $y' = \sqrt{x+y}$ and $x_0 = 0.4$, $y_0 = 0.41$ (take $h = 0.4$). Also calculate y(1.2).

With my best wishes
Dr. Mona Sameeh

Model Answer

Question 1:

(a) $y = mx^c$

$\ln y = \ln m + c \ln x$

$Y = M + cX$

$Y = \ln y, \; M = \ln m, \; X = \ln x$

x	y	$X = \ln x$	$Y = \ln y$	X^2	XY
1	0.12	0	-2.1203	0	0
1.5	0.52	0.4055	-0.6539	0.1644	-0.2651
2	1.47	0.6931	0.3853	0.4805	0.267[illegible]
2.5	3.24	0.9163	1.1756	0.8396	1.0772
3	6.21	1.0986	1.8262	1.2069	2.0062
3.5	10.78	1.2528	2.3777	1.5694	2.9787
4	17.35	1.3863	2.8536	1.9218	3.9559
		5.7526	5.8441	6.1826	10.01[illegible]

$5.8441 = 7M + 5.7526C \rightarrow (1)$

$10.0199 = 5.7526M + 6.1826C \rightarrow (2)$

$M = -2.1115 \Rightarrow m = 0.1211$

and $C = 3.5853$.

$$P(x) \cong \frac{(x-1)(x-3)(x-6)}{(-1)(-3)(-6)}\,2 + \frac{(x-0)(x-3)(x-6)}{(1)(-2)(-5)}\,4$$

$$+ \frac{(x-0)(x-1)(x-6)}{(3)(2)(-3)}\,10 + \frac{(x-0)(x-1)(x-3)}{(6)(5)(3)}\,18$$

$$P(\cdot) \cong \frac{(1)(-1)(-4)}{-18}\,2 + \frac{(2)(-1)(-4)}{10}\,4$$

$$+ \frac{(2)(1)(-4)}{-18}\,10 + \frac{(2)(1)(-1)}{90}\,18$$

$$\cong 6.8$$

(c) $x = \frac{x^3+5}{8} \Rightarrow \phi(x) = \frac{x^3+5}{8}$

$\alpha \in (0,1)$ $\quad \phi'(x) = \frac{3x^2}{8}$

$\Rightarrow |\phi'(x)| < 1 \quad \forall \; x \in (0,1)$

$$\boxed{x_{n+1} = \frac{x_n^3+5}{8}}$$

$x_0 = 0$

$x_1 = 0.625$

$x_2 = 0.655518$

$x_3 = 0.66021$

$x_4 = 0.660971$

$x_5 = 0.661096$

$x_6 = 0.661116$

$x_7 = 0.66112$

$x_8 = 0.66112$ #

Question 2:

1) $x_1^{(k+1)} = \frac{1}{5}\left[x_2^{(k)} - 3x_3^{(k)} - 2\right]$

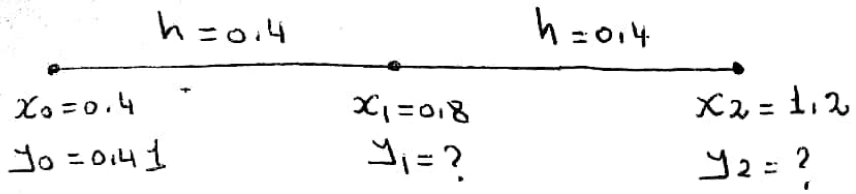
$x_2^{(k+1)} = \frac{1}{5}\left[-x_1^{(k+1)} + 2x_3^{(k)} + 10\right]$

$x_3^{(k+1)} = \frac{1}{10}\left[-2x_1^{(k+1)} + 4x_2^{(k+1)} + 6\right]$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$
0	0	0	0
1	-0,4	2,08	1.512
2	-0,8912	2,78304	1,891456
3	-0.978266	2,952236	1,976547
4	-0.9955481	2.989715	1,994982
5	-0,999046	2,997802	1,998933
6	-0.999798	2,999532	1,999772
7	-0,999957	2,9999	1,999952
8	-0,999991	2,999979	1,99999
9	-0,999998	2,999995	1,999998
10	-1	2,999999	2
11	-1	3	2
12	-1	3	2

$f(x,y) = \sqrt{x+y}$ $\quad x_0 = 0.4$ $\quad y_0 = 0.41$

$h = 0.4$



$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad n = 0, 1.$$

where

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h f(x_n + h, y_n + k_3)$$

$n = 0$

$k_1 = h f(x_0, y_0)$

$= 0.4\sqrt{0.4 + 0.41} = 0.36.$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.4\sqrt{0.6 + 0.59} = 0.4363484844.$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.4\sqrt{0.6 + 0.628174242} = 0.4432992092.$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$= 0.4\sqrt{0.8 + 0.8532992092} = 0.5143216256.$

$$\therefore y(0.8) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 0.8489337963.$$

$n = 1$

$$k_1 = h\, f(x_1, y_1)$$

$$= 0.4\sqrt{0.8 + 0.8489337963}$$

$$= 0.5136432688.$$

$$k_2 = h\, f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 0.4\sqrt{1 + 1.1057554531} = 0.5804488512.$$

$$k_3 = h\, f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.4\sqrt{1 + 1.1391588222} = 0.5850344568.$$

$$k_4 = h\, f(x_1 + h, y_1 + k_3)$$

$$= 0.4\sqrt{1.2 + 1.4339682533} = 0.6491801912.$$

$$y_2 = y(1.2) = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.4312332142.$$