



Answer the following questions

Question 1 [15 points]

(a) Only, make linearization to the curve: $y = 1 + \sin(Ax + B)$.

(b) Based on the least square method, fit $y = be^{ax}$ to the data:

x	0	0.5	1	1.5
y	2	4.5	10	21

(c) Let, $f(x) = \frac{1}{1+x^3}$. Use Lagrange interpolation of the second degree based on nodes $x_0 = 0$, $x_1 = 1$, $x_2 = 2$ to approximate the integral: $\int_0^2 f(x) dx$.

Question 2 [15 points]

(a) Only, find the interval in which the iteration process: $x_{n+1} = 8 + \frac{9}{x_n}$ converge.

(b) Only, reformulate: $x - \tan x = 0$ in the form: $x_{n+1} = \phi(x_n)$ such that the simple iteration method converge

(c) Use Newton-Raphson method to find the root of: $x^3 = 2x + 5$ correct to 6-decimal. Take $x_0 = 2$.

Question 3 [12 points]

(a) Transform the general second order I.V.P $y'' + ay' + by = f(x)$, $y(0) = y_0$, $y'(0) = y_1$ where, a,b,y₀ and y₁ are constants and f(x) is arbitrary function, to its equivalent system of first order equations. Only write the scheme of solution using Rung-Kutta4 method. Illustrate your answer by flowcharts.

(b) Use Rung-Kutta4 method to find x(0.1) and y(0.1) to the first order system: $\dot{x} = 2y + 4t$, $x(0) = 4$, and $\dot{y} = -2y + 4x - 4t - 2$, $y(0) = -5$. If the exact solution is $x(t) = 3e^{-4t} + e^{2t}$, $y(t) = e^{2t} - 6e^{-4t} - 2t$, find the error.

Question 4 [8 points]

(a) Classify the PDE: (i) $5U_{xx} + 6U_{yy} = 0$ (ii) $U_{xy} = 0$ (iii) $U_{xx} - xU_{yy} = 0$.

(b) Use finite difference method to solve Poisson equation $U_{xx} + U_{yy} = 10(x^2 + y^2)$ in the rectangle $0 \leq x \leq 3$, $0 \leq y \leq 2$. Take $h = k = 1$ and the B.C. $U(x, y) = 20x + 30y$, $0 \leq x \leq 3$, $0 \leq y \leq 2$.

Best of luck

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Question (1)

$$(a) \quad y = 1 + \sin(Ax + B)$$

$$y - 1 = \sin(Ax + B)$$

$$\sin^{-1}(y - 1) = Ax + B$$

$$\sin^{-1}(y - 1) = Ax + b$$

b)

1) $y = Ax + B$

$y = be^{Ax}$

x	0	0.5	1	1.5
y	2	4.5	10	21

$\ln y = \ln b + Ax$

$Y = \ln y$

$Y = B + AX$

$B = \ln b = 0.7081$

$b = 2.0301$

x	y	Y	XY	X ²
0	2	0.6931	0	0
0.5	4.5	1.5241	0.7521	0.25
1	10	2.3026	2.3026	1
1.5	21	3.0445	4.5668	2.25
Σ 3		7.5443	7.6215	3.5

$A \Sigma x^2 + B \Sigma x = \Sigma xy$

$A \Sigma x + 4 B = \Sigma y$

$3.5A + 3B = 7.6215$

$3A + 4B = 7.5443$

$A = 1.5706$

$B = 0.7081$

$y = 2.0301 e^{1.5706x}$

(?)

$$c) \quad f(x) = \frac{1}{1+x^3}$$

$$x_0 = 0 \quad x_1 = 1 \quad x_2 = 2$$

x	0	1	2
y	1	1/2	1/9

$$P_2(x) = \frac{(x-1)(x-2)}{(-1)(-2)} x + \frac{(x)(x-2)}{(1)(1-2)} x \frac{1}{2} + \frac{(x)(x-1)}{(2)(2-1)} x \frac{1}{9}$$

$$P_2(x) = (x-1)(x-2) x \frac{1}{2} + (x)(x-2) x - \frac{1}{2} + (x)(x-1) x \frac{1}{18}$$

$$= (x^2 - 2x - x + 2) x \frac{1}{2} + (x^2 - 2x) x - \frac{1}{2} + (x^2 - x) x \frac{1}{18}$$

$$\frac{1}{2}x^2 - \frac{3}{2}x + 1 - \frac{1}{2}x^2 + x - \frac{1}{18}x^2 + \frac{1}{18}x$$

$$P_2(x) = \frac{1}{8}x^2 - \frac{5}{8}x + 1$$

$$P_2(2) = \frac{1}{8}(2)^2 - \frac{5}{8}(2) + 1 = -1$$

$$P_2(0) = 1$$

(3)

Q2)

a) $x_{n+1} = 8 + \frac{9}{x_n}$

$$\phi = 8 + \frac{9}{x} = 8 + 9x^{-1}$$

~~$\phi = 8 + 9x^{-1}$~~ $\phi' = -9x^{-2}$

$$|-9x^{-2}| < 1$$

$$|x^{-2}| > \frac{1}{9}$$

$$|x^2| > 9$$

$$|x| > 3$$

Q2) b) $x - \tan x = 0$

$$x = \tan x$$

$$\phi = x = \tan^{-1}(x)$$

$$\phi' = \frac{1}{1+x^2} \Rightarrow 1+x^2 > 1$$

$$\therefore |\phi'| < 1 \Rightarrow \text{X}$$

$$c) \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow \# \quad f(x) = x^3 - 2x + 5$$

$$f'(x) = 3x^2 - 2$$

$$x_1 = x_0 - \frac{x_0^3 - 2x_0 + 5}{3x_0^2 - 2}$$

n	x_n
1	2,1
2	2,094568
3	2,094551
4	2,094551
5	2,094551

$$\text{Root} = 2,094551$$

Question (3),

$$(a) \quad y'' + ay' + by = f(x)$$

$$y(0) = y_0 \quad x_0 = 0$$

$$y'(0) = y_1 \quad t_0 = 0$$

$$\dot{y} = x \quad \dot{x} + ax + by = f(x)$$

$$\dot{x} = -ax - by$$

$$f_1 = x$$

$$f_2 = -ax - by$$

$$k_1 = hf_1(x_0, y_0, t_0)$$

$$G_1 = hf_2(x_0, y_0, t_0)$$

$$k_2 = hf_1(x_0 + \frac{G_1}{2}, y_0 + \frac{k_1}{2}, t_0 + \frac{h}{2})$$

$$G_2 = hf_2(x_0 + \frac{G_1}{2}, y_0 + \frac{k_1}{2}, t_0 + \frac{h}{2})$$

$$k_3 = hf_1(x_0 + \frac{G_2}{2}, y_0 + \frac{k_2}{2}, t_0 + \frac{h}{2})$$

$$G_3 = hf_2(x_0 + \frac{G_2}{2}, y_0 + \frac{k_2}{2}, t_0 + \frac{h}{2})$$

$$k_4 = hf_1(x_0 + \frac{G_3}{2}, y_0 + \frac{k_3}{2}, t_0 + \frac{h}{2})$$

$$G_4 = hf_2(x_0 + \frac{G_3}{2}, y_0 + \frac{k_3}{2}, t_0 + \frac{h}{2})$$

$$y_i =$$

⑥

$$x = 2y + 4t$$

$$\dot{y} = 2y + 4x - 4t - 2$$

$$x_0 = 4, y_0 = -5$$

$$x(0) = 4$$

$$y(0) = -5$$

$$t_1 = 0.1 \text{ s, } h = t_1 - t_0 = 0.1$$

$$\textcircled{4} F_1(x, y, t) = -2y + 4x - 4t - 2$$

$$K_1 = hf_1(x_0, y_0, t_0) = 0.1 f(4, -5, 0) = -2.4$$

$$K_2 = hf_1(x_0 + \frac{G_1}{2}, y_0 + \frac{K_1}{2}, t_0 + \frac{h}{2}) = 1.94$$

$$K_3 = hf_1(x_0 + G_2, y_0 + \frac{K_2}{2}, t_0 + \frac{h}{2}) = 2.038$$

$$K_4 = hf_1(x_0 + G_3, y_0 + K_3, t_0 + h) = 1.638$$

$$y(t_0 + h) = y(t_1) = y_1 = y(0.1) = y_0 + \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

$$\textcircled{5} F_2(x, y, t) = 2y + 4t$$

$$G_1 = hf_2(x_0, y_0, t_0) = 0.1 f(4, -5, 0) = -1$$

$$G_2 = hf_2(x_0 + \frac{G_1}{2}, y_0 + \frac{K_1}{2}, t_0 + \frac{h}{2}) = -0.74$$

$$G_3 = hf_2(x_0 + \frac{G_2}{2}, y_0 + \frac{K_2}{2}, t_0 + \frac{h}{2}) = -0.786$$

$$G_4 = hf_2(x_0 + G_3, y_0 + K_3, t_0 + h) = -0.5524$$

$$x(t_0 + h) = x(t_1) = x_1 = x(0.1) = x_0 + \frac{1}{6} [G_1 + 2G_2 + 2G_3 + G_4]$$

$$\therefore y_1 = -5 + \frac{1}{6} [2.4 + 2(1.94) + 2(2.038) + 1.638] = \boxed{-3.001}$$

$$\therefore y_{\text{exact}} = \frac{5}{6} e^{2t} - 6e^{-4t} - 2t = \boxed{-3.0005}$$

$$\text{error} = 5 \times 10^{-4}$$

$$\therefore x_1 = 4 + \frac{1}{6} [-1 + 2(-0.74) + 2(-0.786) + (-0.5524)] = \boxed{3.2326}$$

$$x_{\text{exact}} = 3e^{-4t} + e^{2t} = \boxed{3.23236}$$

$$\therefore \text{error} = 2.4 \times 10^{-4}$$

(7)

Q4)

a) Classify

$$(i) 5u_{xx} + 6u_{yy} = 0$$

$$Au_{xx} + Bu_{xy} + Cu_{yy} = 0$$

$$A=5 \quad B=0 \quad C=6$$

$$D = B^2 - 4AC = 0^2 - 4 \times 5 \times 6$$

$$0^2 - 120 = \boxed{-120} \Rightarrow$$

$$D < 0$$

elliptic

$$(ii) u_{xy} = 0$$

$$Au_{xx} + Bu_{xy} + Cu_{yy} = 0$$

$$A=0, B=1, C=0$$

$$D = B^2 - 4AC = 1^2 - 4 \times 0 \times 0 = 1$$

$$D > 0 \Rightarrow \text{hyperbolic}$$

$$(iii) u_{xx} - xu_{yy} = 0$$

$$(Au_{xx} + Bu_{xy} + Cu_{yy} = 0)$$

$$A=1$$

$$C=-x$$

$$B=0$$

$$D = B^2 - 4AC = 0^2 - 4 \times 1 \times (-x) = \boxed{+4x}$$

X		
$\Delta = 0$	$\Delta = +ve$	$\Delta = -ve$
Parabolic	$\Delta > 0$ hyperbolic	$\Delta < 0$ elliptic

6

$$u_{xx} + u_{yy} = 10(x^2 + y^2)$$

$$0 \leq x \leq 3$$

$$0 \leq y \leq 2$$

$$h = k = 1$$

$$u(x, y) = 20x + 30y \quad 0 \leq x \leq 3 \quad 0 \leq y \leq 2$$

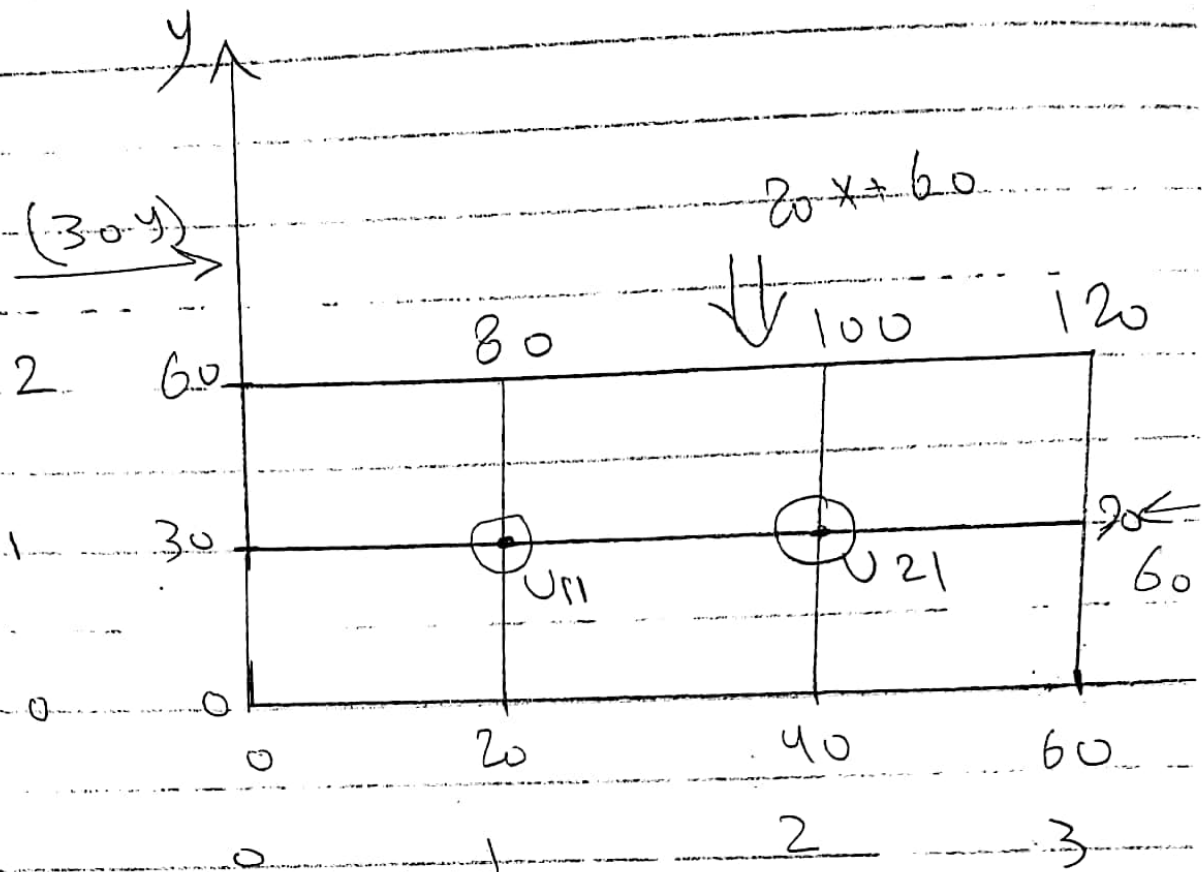
Solu

$$u(0, y) = 20 \cdot 0 + 30y = 30y$$

$$u(x, 0) = 20x + 30 \cdot 0 = 20x$$

$$u(3, y) = 20 \cdot 3 + 30y = 60 + 30y$$

$$u(x, 2) = 20x + 30 \cdot 2 = 20x + 60$$



U11

$$80 + 30 + 20 + U_{21} - 4U_{11} = h^2 f(x)$$

$$130 + U_{21} - 4U_{11} = 1 \times 10 (x^2 + y^2)$$

$$130 + U_{21} - 4U_{11} = 10 (12 + 12)$$

$$130 + U_{21} - 4U_{11} = 20$$

$$U_{21} - 4U_{11} = -110 \rightarrow (1)$$

U21

$$100 + U_{11} + 40 + 90 - 4U_{21} =$$

$$1 \times 10 (2^2 + 12)$$

$$230 + U_{11} - 4U_{21} = 50$$

$$U_{11} - 4U_{21} = -180$$

$$-4U_{21} + U_{11} = -180 \rightarrow (2)$$

کلاس فائنل

Mode 51

$$U_{11} = 55.333$$

$$U_{21} = 41.3333$$