Mansoura University
Faculty of Engineering

Review examples

Biomedical Engineering Programs

Prof. Dr. Magdi S. El-Azab | Numerical Analysis (MTH 201) | Time allowed: 30 min.

Name:

1. [9 marks] Starting with the initial values $\left(x_1^{(0)},\ x_2^{(0)},\ x_3^{(0)},\ x_4^{(0)}\right) = (2,\ 2,\ 5,\ 5)$, solve the following system of equations correct to three decimal places

$$\begin{array}{rcrcrcrcr} 10x_1 & + & x_2 & - & 2x_3 & + & x_4 & = & 10 \\ x_1 & - & 10x_2 & - & x_3 & + & x_4 & = & -20 \\ 2x_1 & - & 2x_2 & + & 10x_3 & - & 4x_4 & = & 50 \\ x_1 & - & x_2 & + & x_3 & - & 10x_4 & = & -40 \end{array}$$

Answer: Solving four equations in four unknowns

	x_1	x_2	x_3	x_4	1
$x_1 =$	0	-0.1	0.2	-0.1	1
$x_2 =$	0.1	0	-0.1	0.1	2
$x_3 =$	-0.2	0.2	0	0.4	5
$x_4 =$	0.1	-0.1	0.1	0	4

Gauss-Seidel method

$k =$	0	2	2	5	5
$k =$	1	1.3000	2.1300	7.1660	4.6336
$k =$	2	1.7568	1.9224	6.8866	4.6721
$k =$	3	1.7179	1.9503	6.9153	4.6683
$k =$	4	1.7212	1.9474	6.9126	4.6686
$k =$	5	1.7209	1.9477	6.9128	4.6686
$k =$	6	1.7209	1.9477	6.9128	4.6686

	x_1	x_2	x_3	x_4	1
$x_1 =$	0	-0.1	0.2	-0.1	1
$x_2 =$	0.1	0	-0.1	0.1	2
$x_3 =$	-0.2	0.2	0	0.4	5
$x_4 =$	0.1	-0.1	0.1	0	4

Gauss-Jacobi method

$k =$	0	2	2	5	5
$k =$	1	1.3000	2.2000	7.0000	4.5000
$k =$	2	1.7300	1.8800	6.9800	4.6100
$k =$	3	1.7470	1.9360	6.8740	4.6830
$k =$	4	1.7129	1.9556	6.9110	4.6685
$k =$	5	1.7198	1.9470	6.9159	4.6668
$k =$	6	1.7218	1.9471	6.9122	4.6689
$k =$	7	1.7208	1.9479	6.9126	4.6687

$k =$	8	1.7209	1.9477	6.9129	4.6686

2. [9 marks] Starting with the initial values $\left(x_1^{(0)},\ x_2^{(0)},\ x_3^{(0)},\ x_4^{(0)}\right) = (1,\ 1,\ 1,\ 1)$, solve the following system of equations correct to three decimal places.

$$\begin{aligned} 10x_1 - 2x_2 + 2x_3 + x_4 &= -30 \\ 4x_1 + 10x_2 - 2x_3 + 2x_4 &= 40 \\ 2x_1 - 2x_2 + 5x_3 - x_4 &= 10 \\ 2x_1 - x_2 + x_3 - 10x_4 &= -20 \end{aligned}$$

<u>**Answer:**</u>

	x1	x2	x3	x4	1
x1 =	0	0.2	-0.2	-0.1	-3
x2 =	-0.4	0	0.2	-0.2	4
x3 =	-0.4	0.4	0	0.2	2
x4 =	0.2	-0.1	0.1	0	2

		Gauss-Seidel method			
k=	0	1	1	5	5
k=	1	-4.3000	5.7200	7.0080	1.2688
k=	2	-3.3845	6.5016	6.2082	1.2938
k=	3	-3.0707	6.2112	5.9715	1.3619
k=	4	-3.0883	6.1572	5.9706	1.3637
k=	5	-3.0991	6.1610	5.9768	1.3618
k=	6	-3.0993	6.1627	5.9772	1.3616

	x1	x2	x3	x4	1
x1 =	0	0.2	-0.2	-0.1	-3
x2 =	-0.4	0	0.2	-0.2	4
x3 =	-0.4	0.4	0	0.2	2
x4 =	0.2	-0.1	0.1	0	2

		Gauss-Jacobi method			
k=	0	1	1	5	5
k=	1	-4.3000	3.6000	3.0000	2.6000
k=	2	-3.1400	5.8000	5.6800	1.0800
k=	3	-3.0840	6.1760	5.7920	1.3600
k=	4	-3.0592	6.1200	5.9760	1.3448
k=	5	-3.1057	6.1499	5.9406	1.3738
k=	6	-3.0955	6.1556	5.9770	1.3579
k=	7	-3.1001	6.1620	5.9720	1.3630
k=	8	-3.0983	6.1618	5.9774	1.3610

1. Given the initial value problem:

$$y' = \sqrt{x} + \sqrt{y}, \quad y(0.16) = 1.44,$$

(i) Evaluate $y(0.2)$ by Runge-Kutta approximation of order 4,
(ii) Find $y(0.12)$ by the use of Taylor method,
(iii) Find $y(0.24)$ by the use of Euler method.

Answer:

(i) Runge-Kutta 4 method y at $x_1 = 0.2$ *take* $x_0 = 0.16 \Rightarrow h = 0.2 - 0.16 = 0.04,$

$$y(0.2) = y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

0.16	0.18	0.2
1.44	1.472	1.505513
	1.47275	

$$k_1 = 0.04f(0.16, 1.44) = 0.064,$$
$$k_2 = 0.04f(0.18, 1.472) = 0.065501,$$
$$k_3 = 0.04f(0.18, 1.47275) = 0.065513,$$
$$k_4 = 0.04f(0.2, 1.505513) = 0.066968$$

$$\boxed{y(0.2) = 1.44 + \frac{1}{6}(0.064 + 2 * 0.065501 + 2 * 0.065513 + 0.066968) = 1.505499}$$

(ii) Taylor method y at $x_1 = 0.12$ *take* $x_0 = 0.16 \Rightarrow h = 0.12 - 0.16 = -0.04,$

$$y' = \sqrt{x} + \sqrt{y} \qquad y_0' = 1.6$$

$$y'' = \frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} \qquad y_0'' = 1.916667$$

$$y''' = \frac{1}{4}x^{-3/2} + \frac{1}{2}\frac{\sqrt{y}y'' - ((y')^2/2\sqrt{y})}{y} \qquad y_0''' = -0.32331$$

$$\boxed{y(0.12) = 1.44 - 0.04(1.6) + \left(\frac{(-0.04)^2}{2}\right)(1.916667) + \left(\frac{(-0.04)^3}{6}\right)(-0.32331) = 1.377537}$$

(iii) Euler method y at $x_1 = 0.24$ *take* $x_0 = 0.2,\ y_0 = 1.505499 \Rightarrow h = 0.24 - 0.2 = -0.04,$

$$\boxed{y(0.24) = 1.505499 + 0.04f(0.2,\ 1.505499) = 1.572468}$$

2. Given the initial value problem:

$$y' + \frac{2x + y^2}{xy} = 0, \quad y(1.2) = 1,$$

(i) Evaluate $y(1.4)$ by Runge-Kutta approximation of order 4,
(ii) Find $y(1)$ by the use of Taylor method,
(iii) Find $y(1.6)$ by the use of Euler method.
(iv) Use a predictor-corrector method to evaluate $y(1.8)$.

Answer:

$$y' = f(x, y) = -\frac{2x + y^2}{xy}, \quad y(1.2) = 1$$

(i) Runge-Kutta 4 method y at $x_1 = 1.4$ *take* $x_0 = 1.2 \;\Rightarrow\; h = 1.4 - 1.2 = 0.2,$

$$y(1.4) = y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

x =	1.2	1.3	1.4
y =	1	0.716667	0.581215
		0.828352	

k1 =	-0.56667	k2 =	-0.3433	k4 =	-0.26053
		k3 =	-0.41879		

$$\boxed{y(1.4) = y(1) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + 2k_4) = 0.608106}$$

(ii) Taylor method y at $x_1 = 1$ *take* $x_0 = 1.2 \;\Rightarrow\; h = -0.2,$

$$y' = -\frac{2x + y^2}{xy} = -\frac{2}{y} - \frac{y}{x} \qquad y_0' = -2.83333$$

$$y'' = \frac{2y'}{y^2} + \frac{y}{x^2} - \frac{y'}{x} \qquad y_0'' = -2.72685$$

$$y''' = \frac{2y''}{y^2} - \frac{4(y')^2}{y^3} + \frac{2y'}{x^2} - \frac{2y}{x^3} - \frac{y''}{x} \qquad y_0''' = -40.385$$

$$\boxed{y(1) = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots = 1.584155}$$

(iii) Euler method to get $y(1.6)$. Take $x_0 = 1.4,\;\; y_0 = 0.608106 \Rightarrow h = 0.2$

$$\boxed{y(1.6) = y_0 + hy_0' = 1.000712}$$

(iv) Predictor-corrector methods to get $y(1.8)$.

	1	-5	19	9
-9	37	-59	55	
1.58416	1	0.60811	1.00071	1.5505
1	1.2	1.4	1.6	1.8
2.25863	2.09545	1.96303	2.26527	2.58683

y1* = 1.550501938 and y1c = 1.489060613

$$y^p(1.8) = y_0 + \frac{h}{24}(55f_3 - 59f_2 + 37f_1 - 9f_0) = 0.9685,$$

$$y^c(1.8) = y_0 + \frac{h}{24}(9f_4 + 19f_3 - 5f_2 + f_0) = 0.5215$$

3. Given the initial value problem

$$y' = \frac{y}{x} + x - 1, \quad y(1) = 3,$$

(i) Evaluate $y(1.3)$ by Runge-Kutta approximation of order 4,

(ii) Find $y(1.5)$ by the use of Taylor method of order 3.

4. Given the initial value problem:

$$y' = \sqrt{x^2 + 4y^2}, \quad y(2.4) = 2,$$

(i) Evaluate $y(2.6)$ by Runge-Kutta approximation of order 4,

(ii) Find $y(2.2)$ by the use of Taylor method,

(iii) Find $y(2.8)$ by the use of Euler method.

(iv) Use the Adams-Bashforth/Adams-Moulton method to evaluate $y(3)$.