Mansoura University
Faculty of Engineering

Midterm Exam.
Tuesday 27/11/2018

Biomedical Engineering Programs

Dr. Magdi S. El-Azab | **Numerical Analysis (MTH 201)** | **Time allowed: 60 min.**

Name:

---

1. Given the table of readings:

x	0.5	1	1.5	2	2.5
y	0.40	0.56	0.82	1.26	1.98

(*a*)[5 pts] Fit these readings for the curve $y = ae^{x} + b$, with the computation of the $L_2 - norm$ of the error.

(*b*)[5 pts] By the use of two different interpolation formulas, find $y(1.25)$ and $y(1.75)$, respectively.

2. [10 pts] Use two different methods to find the smallest positive root of the equation $e^{x} - 3x^{2} - 1 = 0$, correct to five decimal places.

Mansoura University
Faculty of Engineering

Midterm Exam.
Tuesday 27/11/2018

Biomedical Engineering Programs

Dr. Magdi S. El-Azab	**Numerical Analysis (MTH 201)**	**Time allowed: 60 min.**

Name:

---

1. Given the table of readings:

x	0.5	1	1.5	2	2.5
y	0.40	0.56	0.82	1.26	1.98

(a)[5 pts] Fit these readings for the curve $y = ae^x + b$, with the computation of the $L_2 - norm$ of the error.

(b)[5 pts] By the use of two different interpolation formulas, find $y(1.25)$ and $y(1.75)$, respectively.

2. [10 pts] Use two different methods to find the smallest positive root of the equation $e^x - 3x^2 - 1 = 0$, correct to five decimal places.

Model answer

1. (a). To fit the given curve to the given readings, we write it as

$Y = AX + B$	$X = e^x, \quad Y = y$
	$A = a, \quad B = b$

So that the normal equations shall be

$$A\sum_{r=1}^{n} X_r^2 + B\sum_{r=1}^{n} X_r = \sum_{r=1}^{n} X_r Y_r,$$

$$A\sum_{r=1}^{n} X_r + 5B = \sum_{r=1}^{n} Y_r$$

Construct the table

x	y	$X = e^x$	$Y = y$	X^2	XY	y approx	d^2
0.5	0.40	1.648721	0.40	2.718282	0.659489	0.398473	2.33E-06
1	0.56	2.718282	0.56	7.389056	1.522238	0.559014	9.72E-07
1.5	0.82	4.481689	0.82	20.08554	3.674985	0.823702	1.37E-05
2	1.26	7.389056	1.26	54.59815	9.310211	1.260097	9.47E-09
2.5	1.98	12.18249	1.98	148.4132	24.12134	1.979592	1.66E-07
Σ		28.42024	5.02	233.2042	39.28826		1.72E-05

Substitute in the normal equations we obtain

$$233.2042A + 28.42024B = 39.28826,$$
$$28.42024A + 5B = 5.02$$

$$a = A = 0.1501, \quad b = B = 0.1510, \quad L_2 - norm\ of\ \text{the}\ error = 0.004145$$

From which we have

$$\boxed{y = 0.1501e^x + 0.1510}$$

(b). $y(1.25)$ and $y(1.75)$

x	y	Δ	Δ^2	Δ^3	Δ^4
0.5	0.40				
		0.16			
1	0.56		0.1		
		0.26		0.08	
1.5	0.82		0.18		0.02
		0.44		0.1	
2	1.26		0.28		
		0.72			
2.5	1.98				

Gauss backward & forward formula							
x0 =	**1.5**	**x =**	**1.25**	h =	0.5	s =	-0.5
	y (	**1.25**	) =	backward	forward		
		three terms		0.6675	0.6675		
		four terms		0.6725	0.67375		
		five terms		0.672969	0.673906		

Gauss forward formula							
xn =	**1.5**	**x =**	**1.75**	h =	0.5	s =	0.5
	y (	**1.75**	) =				
		three terms		1.0175			
		four terms		1.01125			
		five terms		1.010781			

Lagrang's interpolation formula (five points)					
x	0.5	1	1.5	2	2.5
y	0.40	0.56	0.82	1.26	1.98
l =	0.023438	-0.15625	0.703125	0.46875	-0.03906
	-0.03906	0.46875	0.703125	-0.15625	0.023438

y (	**1.75**	**) =**	1.01171875
y (	**1.25**	**) =**	0.67296875

2.

$$f(x) = e^x - 3x^2 - 1$$

f(	0	)	=	0
f(	0.25	)	=	0.09652542
f(	0.5	)	=	-0.1012787
f(	0.75	)	=	-0.5705

There is at least one root in the interval (0.25, 0.5). So

False Position method		
n	x	f(x)
	0.25	0.09652542
	0.5	-0.1012787
1	0.371996	0.03548411
2	0.405208	0.00703382
3	0.411364	0.00121347
4	0.412413	0.00020468
5	0.412590	3.3977E-05
6	0.412619	5.9957E-06
7	0.412624	1.1709E-06
8	0.412625	2.0593E-07
9	0.412625	2.0593E-07

Newton-Raphson method	
n	x
1	0.25
2	0.696930
3	0.490143
4	0.422750
5	0.412852
6	0.412625
7	0.412625