Mansoura University Faculty of Engineering Biomedical Engineering	3D Modeling November 2017	Mid Term Exam Level 300 Time allowed 1 Hr

1. A storage tank contains a liquid at depth y where y = 0 when the tank is half full. Liquid is withdrawn at a constant flow rate Q to meet demands. The contents are resupplied at a sinusoidal rate $3Q\ \sin^2(t)$. Assume that the density of the liquid is constant. Apply the conservation of mass law to obtain

i. dV/dt.

ii. dy/dt where V =Ay.

2. Couette flow (see Figure 1) is frequently used in undergraduate physics and engineering courses to illustrate shear-driven fluid motion. The simplest conceptual configuration finds two infinite, parallel plates separated by a distance h. One plate, the top one, translates with a constant velocity U in its own plane. Neglecting pressure gradients, the Navier–Stokes equations simplify to

$$\frac{d^2u}{dy^2}=0$$

where y is a spatial coordinate normal to the plates and u (y) is the velocity distribution. Determine.

i. Integrate the above equation 2 times with respect to y. Write the two boundary conditions required to determine the constants C_1, C_2.

ii. By solving these two equations, obtain the values of constants C_1, C_2.

iii. Why the number of boundary conditions cannot be lower than two.

iv. Why the number of boundary conditions cannot be higher than two.

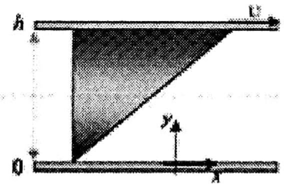
v. Determine the volumetric flow rate (Q). Take the plate width = b.

vi. Determine the mean velocity (V).

vii. Obtain the velocity distribution equation as a function of the following parameters: the mean velocity (V), the plate height (h), and the y-axis.

viii. Where does the velocity distribution, u(y), equal the mean velocity (V)?

ix. Does the shear stress change with the variation of the y-axis?. Why?



3. Complete the following sentences:

i. .... is an example of fluid with high Prandtl number (Pr). The Prandtl number (Pr) is defined as ....

ii. Each term in the momentum equation has the units of ....

iii. According to .... law, the energy flow term by conduction in the inlet z direction equals .... This term in the energy equation has the units of ....

iv. The energy flow term by convection in the outlet x direction equals .... + ...

Good Luck

Dr. M. M. Awad

BME

Mansoura University
Faculty of Engineering
Biomedical Engineering

3D Modeling
November 2017

Mid Term Exam
Level 300
Time allowed 1 Hr

Question 1:

Apply the conservation of mass law, we have

$m_{stored} = m_{in} - m_{out}$

or

$d(\rho V)/dt = \rho Q_{in} - \rho Q_{out} = \rho(Q_{in} - Q_{out})$

The density of the liquid is constant.

$\rho dV/dt = \rho(Q_{in} - Q_{out})$

Divide both sides by ρ, we obtain

$dV/dt = Q_{in} - Q_{out}$

$Q_{in} = 3Q \sin^2(t)$

$Q_{out} = Q$

$dV/dt = 3Q \sin^2(t) - Q$

$V = Ay$

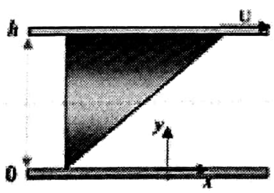
$dV/dt = d(Ay)/dt = 3Q \sin^2(t) - Q$

The surface area is constant.

$d(Ay)/dt = A(dy/dt) = 3Q \sin^2(t) - Q$

$dy/dt = (3Q \sin^2(t) - Q)/A$

Question 2:



i. $\dfrac{d^2u}{dy^2} = 0$

Integrate the above equation with respect to y, we obtain
$du/dy = C_1$
Integrate the above equation with respect to y, we obtain

$u(y) = C_1 y + C_2$
Boundary Condition 1, at $y = 0$, $u(y) = 0$ (i)
Boundary Condition 2, at $y = h$, $u(y) = U$ (ii)
ii. From Boundary Condition 1, $C_2 = 0$
ii. From Boundary Condition 2, $U = C_1 h$
$C_1 = U/h$
$u(y) = Uy/h$
iii. If the number of boundary conditions are lower than two, we have one equation and two unknowns. In this case, the number of unknowns exceeds the number of equations. Then, the system must be inconsistent. i.e. There is no solution.
iv. If the number of boundary conditions are higher than two, we have three equations (for example) and two unknowns. Thus, the number of equations in a linear system exceeds the number of unknowns. Then, a homogeneous system is guaranteed to have infinitely many solutions. i.e. There would be infinite possible solutions to these equations.
v. $Q = \int u(y)\, dA$
$u(y) = Uy/h$
$dA = b dy$
$Q = \int (Uby/h)\, dy$
$Q = (Ub/h) \int y\, dy$
$Q = (Ub/h) * (y^2/2)$ for $y = 0$ to h
$Q = (Ub/h) * (h^2/2)$
$Q = Ubh/2$
vi. $Q = VA$
$Q = Ubh/2$
$A = bh$
$Q = UA/2$
$V = U/2$
vii. $u(y) = Uy/h$
$V = U/2$
$U = 2V$
$u(y) = 2Vy/h$
viii. When $u(y) = V$
$2Vy/h = V$
$2y/h = 1$
$y = h/2$ (i.e. at the half plate height)
ix. $\tau = -\mu du/dy$
$du/dy = C_1 = U/h$
$\tau = -\mu U/h$
Thus, the shear stress does not change with the variation of the y-axis.

Question 3:

i. Oil is an example of fluid with high Prandtl number (Pr). The Prandtl number (Pr) is defined as ratio of momentum diffusivity (ν) to thermal diffusivity (α).
ii. Each term in the momentum equation has the units of Newton.
iii. According to Fourier law, the energy flow term by conduction in the inlet z direction equals $-k dx dy (\partial T/\partial z)$. This term in the energy equation has the units of Watt.
iv. The energy flow term by convection in the outlet x direction equals $(\rho u dy dz)h + (\partial((\rho u dy dz)h)/\partial x)dx$

BME