Mansoura University Faculty of Engineering Dept. of Power Mech. Eng. Course Title: 3D Modeling & Simulation of Heat Transfer & Fluid Flow Course Code: MPE371	BME Level 300 December 2017 Exam Type: Final Time: 2 Hours Full Mark: 50

Attempt All Questions. Assume Any Missing Data. Questions Are Equal Weight But Are Not Equal Difficulty.

Question (1)

1.a Define the momentum. What is its units?

1.b Define the Reynolds number (Re).

1.c Why do we put the negative sign in the law of shear stress (τ)?. When does the shear stress (τ) equal zero?

1.d What is the difference between ordinary differential equations (ODEs) and partial differential equations (PDEs)?

1.e Mention the names of three types of partial differential equations (PDEs). Explain how can we classify the partial differential equations (PDEs) into three categories.

Question (2)

The velocity distribution for the flow of a Newtonian fluid between two infinite, parallel plates separated by a distance h is shown in Figure 1.The x momentum equation simplifies to

$$\mu \frac{d^2u}{dy^2} = -\rho g_x$$

where y is a spatial coordinate normal to the plates, u (y) is the velocity distribution, and g_x is the gravity component in the x direction. Determine.

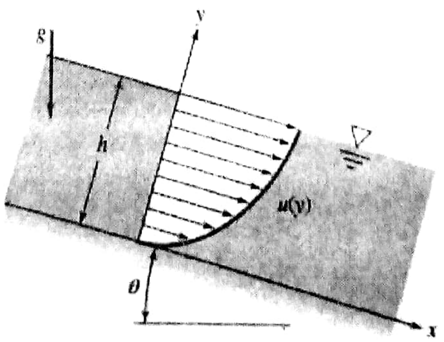
i. Integrate the above equation 2 times with respect to y. Use the two boundary conditions (no-slip condition at the solid surface and zero shear stress at the liquid free surface) to determine the constants C_1, C_2.

ii. By solving these two equations, obtain the values of constants C_1, C_2.

iii. Determine the volumetric flow rate (Q). Take the plate width = b.

iv. Determine the mean velocity (V).

v. Obtain the velocity distribution equation as a function of the following parameters: the mean velocity (V), the plate height (h), and the y-axis.



Question (3)

For the following velocity distribution.

$$u/U_\infty = a + b\eta$$

Where a, b are constants and $\eta = y/\delta$

i. Write the two boundary conditions required to determine the constants a, b.

ii. By solving these two equations, obtain the values of constants a, b.

iii. Calculate the displacement thickness (δ^*) and momentum thickness (θ) in terms of δ.

iv. Calculate the dimensionless profile shape factor ($H = \delta^*/\theta$).

Question (4)

Consider the steady-state temperature distribution for a long, thin rod positioned between two constant-temperature walls. The rod's cross sectional dimensions are small enough so that radial temperature gradients are minimal and, consequently, temperature is a function exclusively of the axial coordinate x. Heat is transferred along the rod's longitudinal axis by conduction and between the rod and the surrounding gas by convection. Radiation is assumed to be negligible. Using a heat balance for a differential element of a heated rod subject to conduction and convection, the differential equation can be expressed as

$$\frac{d^2T}{dx^2} + h'(T_\infty - T) = 0$$

where h' = a bulk heat-transfer parameter reflecting the relative impacts of convection and conduction = 2h/(rk).

i. Obtain the units of h' in 2 different ways.

ii. Show that

$$\frac{d^2T}{dx^2} \cong \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2}$$

iii. Use the finite-difference approach to get the 4 x 4 matrix for a 10-m rod with h' = 0.02, T_∞ = 20°C, and the boundary conditions: T (0) = 40°C, T (10) = 200°C. Use four interior nodes with a segment length of Δx = 2 m.

Question (5)

Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of T_0 = 100°C and T_5 = 420°C respectively. The one-dimensional problem is governed

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

Take rod length L equals 0.4 m, Thermal conductivity k equals 1000 W/m.K, cross-sectional area A is 10×10^{-3} m^2, δx = 0.1 m.

i. Integrate the above equation 2 times with respect to x. Use the 2 boundary conditions at rod ends to obtain the values of the constants C_1 & C_2. Use this temperature distribution equation to obtain the values of the temperature T_1, T_2, T_3, T_4 at x = 0.05, 0.15, 0.25, 0.35 m respectively.

ii. Using the control volume method, obtain the discretised equation for nodal points 2 and 3, the discretised equation for nodal point 1, and the discretised equation for nodal point 4.

Good Luck

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Faculty of Engineering
Biomedical Engineering

3D Modeling
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Final Exam
Level 300
Time allowed 2 Hrs

Question 1:

1.a The momentum equals mass times velocity.
momentum = mass * velocity
Its units is kg.m/s

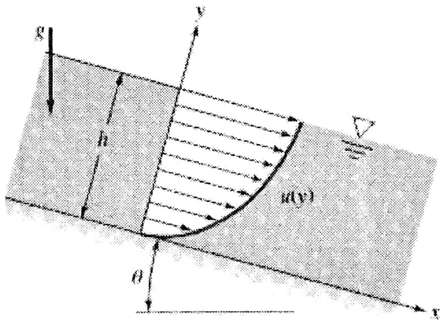
1.b The Reynolds number (Re) is the ratio of inertia force to viscous force.
$Re = \rho UD/\mu$

1.c We put the negative sign in the law of shear stress (τ)
$\tau = -\mu * dU/dy$
because the slope of the velocity gradient (dU/dy) is negative.
The shear stress (τ) equals zero when the velocity is maximum (i.e. the velocity gradient (dU/dy) is zero)

1.d ordinary differential equations (ODE) have functions of one variable.
partial differential equations (PDE) have functions of multiple variables.

1.e Three types of partial differential equations (PDEs): Elliptic, Parabolic, Hyperbolic.
For $Au_{xx}+Bu_{yx}+C\,u_{yy}+D = 0$
$B^2-4AC < 0$, PDE is Elliptic
$B^2-4AC = 0$, PDE is Parabolic
$B^2-4AC > 0$, PDE is Hyperbolic

Question 2:



i. $\mu \dfrac{d^2u}{dy^2} = -\rho g_x$

Rearranging the above equation, we have

$$\frac{d^2u}{dy^2} = -\frac{\rho}{\mu} g_x = -\frac{\rho}{\mu} g \sin\theta$$

Integrate the above equation with respect to y, we obtain
$du/dy = (-\rho g \sin\theta/\mu)y + C_1$
Integrate the above equation with respect to y, we obtain
$u(y) = (-\rho g \sin\theta/2\mu)y^2 + C_1 y + C_2$
Boundary Condition 1, at $y = 0$, $u(y) = 0$ (i)
Boundary Condition 2, at $y = h$, $du/dy = 0$ (ii)
ii. From Boundary Condition 1, $C_2 = 0$
ii. From Boundary Condition 2, $0 = (-\rho g \sin\theta/\mu)h + C_1$
$C_1 = (\rho g \sin\theta/\mu)h$

$u(y) = (-\rho g\sin\theta/2\mu)y^2 + (\rho g\sin\theta/\mu)hy = (\rho g\sin\theta/\mu)h^2[(y/h)-0.5(y/h)^2]$

iii. $Q = \int u(y)\, dA$

$u(y) = (\rho g\sin\theta/\mu)h^2[(y/h)-0.5(y/h)^2]$

$dA = bdy$

$Q = \int (\rho g\sin\theta/\mu)bh^2[(y/h)-0.5(y/h)^2]\, dy$

$Q = (\rho g\sin\theta/\mu)bh^2 \int [(y/h)-0.5(y/h)^2]\, dy$ for $y = 0$ to h

$Q = (\rho g\sin\theta/\mu)bh^2 * [(y^2/2h)-(y^3/6h^2)]$ for $y = 0$ to h

$Q = (\rho g\sin\theta/\mu)bh^2 * [(h/2)-(h/6)]$

$Q = (\rho g\sin\theta/\mu)bh^2 * [(h/3)]$

$Q = (\rho g\sin\theta/3\mu)bh^3$

iv. $Q = VA$

$Q = (\rho g\sin\theta/3\mu)bh^3$

$A = bh$

$V = (\rho g\sin\theta/3\mu)h^2$

v. $u(y) = (-\rho g\sin\theta/2\mu)y^2 + (\rho g\sin\theta/\mu)hy = (\rho g\sin\theta/\mu)h^2[(y/h)-0.5(y/h)^2]$

$V = (\rho g\sin\theta/3\mu)h^2$

$u(y) = 3V[(y/h)-0.5(y/h)^2]$

Question 3:

$$u/U_\infty = a + b\eta$$

i. Boundary Condition 1, at $y = 0$ ($\eta = 0$), $u/U_\infty = 0$ (i)

Boundary Condition 2, at $y = \delta$ ($\eta = 1$), $u/U_\infty = 1$ (ii)

ii. From Boundary Condition 1, $a = 0$

ii. From Boundary Condition 2, $b = 1$

$$u/U_\infty = \eta$$

iii. The displacement thickness (δ^*) $= \delta \int(1- u/U_\infty)\, d\eta$ for η from 0 to 1

$\delta^* = \delta \int(1- \eta)\, d\eta$ for η from 0 to 1

$\delta^* = \delta*(\eta-0.5\, \eta^2)$ for η from 0 to 1

$\delta^* = \delta/2$

iii. The momentum thickness (θ) $= \delta \int u/U_\infty*(1- u/U_\infty)\, d\eta$ for η from 0 to 1

$\theta = \delta \int \eta(1- \eta)\, d\eta$ for η from 0 to 1

$\theta = \delta \int (\eta- \eta^2)\, d\eta$ for η from 0 to 1

$\theta = \delta*(0.5\, \eta^2-0.333\, \eta^3)$ for η from 0 to 1

$\theta = \delta/6$

iv. The dimensionless profile shape factor ($H = \delta^*/\theta$) $= (\delta/2)/(\delta/6) = 3$

Question 4:

i. Units of $h' = m^{-2}$

iii. $T_{i-1} + T_{i+1} + 0.02*20*\Delta x^2 = 2T_{i-} + 0.02*\Delta x^2*T_{i-}$

$T_{i-1} + T_{i+1} + 1.6 = 2T_{i-} + 0.08T_{i-} = 2.08T_i$

For node 1, $40 + T_2 + 1.6 = 2.08T_1$ (i.e. $2.08T_1 - T_2 = 41.6$)

For node 2, $T_1 + T_3 + 1.6 = 2.08\, T_2$ (i.e. $- T_1 + 2.08T_2 - T_3 = 1.6$)

For node 3, $T_2 + T_4 + 1.6 = 2.08\, T_3$ (i.e. $- T_2 + 2.08T_3 - T_4 = 1.6$)

For node 4, $T_3 + 200 + 1.6 = 2.08\, T_4$ (i.e. $- T_3 + 2.08T_4 = 201.6$)

Question 5:

i. $\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0$

Since the thermal conductivity (k) is constant, we have

$k\frac{d^2T}{dx^2}=0$

Since the thermal conductivity (k) is constant, we have

$\frac{d^2T}{dx^2}=0$

Integrate the above equation with respect to x, we obtain
$dT/dx = C_1$
Integrate the above equation with respect to x, we obtain
$T(x) = C_1x + C_2$
Boundary Condition 1, at x = 0, T(x) = 100 (i)
Boundary Condition 2, at x = 0.4, T(x) = 420 (ii)
ii. From Boundary Condition 1, $C_2 = 100$
ii. From Boundary Condition 2, $C_1 = 800$
T(x) = 800x + 100
at x = 0.05, T(0.05) = 800*0.05 + 100 = 140
at x = 0.15, T(0.15) = 800*0.15 + 100 = 220
at x = 0.25, T(0.25) = 800*0.25 + 100 = 300
at x = 0.35, T(0.35) = 800*0.35 + 100 = 380

The discretised equation for nodal point 2

$2T_2 = T_1 + T_3$

Check: 2*220 = 440 = 140 + 300

The discretised equation for nodal point 3

$2T_3 = T_2 + T_4$

Check: 2*300 = 600 = 220 + 380

The discretised equation for nodal point 1

$3T_1 = T_2 + 200$

Check: 3*140 = 420 = 220 + 200

The discretised equation for nodal point 4

$3T_4 = T_3 + 840$

Check: 3*380 = 1140 = 300 + 840