

1. An idealized incompressible flow has the proposed three-dimensional velocity distribution  $V = xy^2\mathbf{i} + f(y)\mathbf{j} - zy^2\mathbf{k}$ . Find the appropriate form of the function  $f(y)$  that satisfies the continuity relation.

2. Two infinite, parallel plates separated by a distance  $h$ . One plate, the bottom one, translates with a constant velocity  $U$  in its own plane while the upper plate is kept fixed. Neglecting pressure gradients, the Navier–Stokes equations simplify to

$$\frac{d^2u}{dy^2} = 0$$

where  $y$  is a spatial coordinate normal to the plates and  $u(y)$  is the velocity distribution. Determine.

- Integrate the above equation 2 times with respect to  $y$ . Write the two boundary conditions required to determine the constants  $C_1, C_2$ .
- By solving these two equations, obtain the values of constants  $C_1, C_2$ .
- Why the number of boundary conditions cannot be lower than two.
- Why the number of boundary conditions cannot be higher than two.
- Determine the volumetric flow rate ( $Q$ ). Take the plate width =  $b$ .
- Determine the mean velocity ( $V$ ).
- Obtain the velocity distribution equation as a function of the following parameters: the mean velocity ( $V$ ), the plate height ( $h$ ), and the  $y$ -axis.
- Where does the velocity distribution,  $u(y)$ , equal the mean velocity ( $V$ )?
- Does the shear stress change with the variation of the  $y$ -axis?. Why?

3.i Complete the following sentence:

According to ..... law, the energy flow term by conduction in the inlet  $x$  direction equals .....  
This term in the energy equation has the units of .....

3.ii. Define the following: the displacement thickness ( $\delta^*$ ), the momentum thickness ( $\theta$ ), the dimensionless profile shape factor ( $H$ ).

3.iii. What is the range of the dimensionless profile shape factor ( $H$ )? Why?

Good Luck  
Dr. M. M. Awad

# mid-term model ans.

Mansoura University  
Faculty of Engineering  
Biomedical Engineering

3D Modeling  
November 2018

Mid Term Exam  
Level 300  
Time allowed 1 Hr

Question 1:

For an idealized incompressible flow has the proposed three-dimensional velocity distribution, the continuity relation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

For the proposed three-dimensional velocity distribution  $V = xy^2\mathbf{i} + f(y)\mathbf{j} - zy^2\mathbf{k}$ , we have

$$u = xy^2, v = f(y), w = -zy^2\mathbf{k}$$

Therefore,  $\partial u / \partial x = y^2$ ,  $\partial w / \partial z = -y^2$ . Substitute these derivatives in Eq. (1), we have

$$y^2 + (\partial f(y) / \partial y) - y^2 = 0$$

$$\partial f(y) / \partial y = 0$$

$$f(y) = \int 0 \, dy = \text{constant}$$

Question 2:

$$i. \frac{d^2 u}{dy^2} = 0$$

Integrate the above equation with respect to y, we obtain

$$du/dy = C_1$$

Integrate the above equation with respect to y, we obtain

$$u(y) = C_1 y + C_2$$

Boundary Condition 1, at  $y = 0$ ,  $u(y) = U$  (i)

Boundary Condition 2, at  $y = h$ ,  $u(y) = 0$  (ii)

ii. From Boundary Condition 1,  $C_2 = U$

ii. From Boundary Condition 2,  $0 = C_1 h + U$

$$C_1 = -U/h$$

$$u(y) = -Uy/h + U = U[1 - (y/h)]$$

iii. If the number of boundary conditions are lower than two, we have one equation and two unknowns. In this case, the number of unknowns exceeds the number of equations. Then, the system must be inconsistent. i.e. There is no solution.

iv. If the number of boundary conditions are higher than two, we have three equations (for example) and two unknowns. Thus, the number of equations in a linear system exceeds the number of unknowns. Then, a homogeneous system is guaranteed to have infinitely many solutions. i.e. There would be infinite possible solutions to these equations.

$$v. Q = \int u(y) \, dA$$

$$u(y) = U[1 - (y/h)]$$

$$dA = b \, dy$$

$$Q = \int U b [1 - (y/h)] \, dy$$

$$Q = (Ub) \int [1 - (y/h)] \, dy$$

$$Q = (Ub) * [y - (y^2/2h)] \text{ for } y = 0 \text{ to } h$$

$$Q = (Ub) * [h - (h^2/2h)]$$

$$Q = Ubh/2$$

$$vi. Q = VA$$

$$Q = Ubh/2$$

$$A = bh$$

$$Q = UA/2$$

$$V = U/2$$

$$\text{vii. } u(y) = -Uy/h + U = U[1-(y/h)]$$

$$V = U/2$$

$$U = 2V$$

$$u(y) = 2V [1-(y/h)]$$

$$\text{viii. When } u(y) = V$$

$$2V [1-(y/h)] = V$$

$$[1-(y/h)] = 1/2$$

$$y/h = 1/2$$

$$y = h/2 \text{ (i.e. at the half plate height)}$$

$$\text{ix. } \tau = -\mu du/dy$$

$$du/dy = C_1 = -U/h$$

$$\tau = \mu U/h$$

Thus, the shear stress does not change with the variation of the y-axis.

Question 3:

i. According to Fourier law, the energy flow term by conduction in the inlet x direction equals  $-kdydz(\partial T/\partial x)$ . This term in the energy equation has the units of Watt.

ii. The displacement thickness ( $\delta^*$ ) =  $\delta \int (1 - u/U_\infty) d\eta$  for  $\eta$  from 0 to 1

The momentum thickness ( $\theta$ ) =  $\delta \int u/U_\infty (1 - u/U_\infty) d\eta$  for  $\eta$  from 0 to 1

The dimensionless profile shape factor ( $H$ ) =  $\delta^*/\theta$

iii. The range of the dimensionless profile shape factor ( $H$ ) is  $H > 1$  because the displacement thickness ( $\delta^*$ ) is always greater than the momentum thickness ( $\theta$ ).