

	Mansoura University	CSE 397: Pattern Recognition	
	Faculty of Engineering	Spring-2018: Self Study	
	Biomedical Eng. Program	Exam date: May 3 rd , 2018	
	Full Mark is 50	Time Allowed is 120 Minutes	
	Exam is FIVE Questions and ONE page. Please attempt ALL questions Assume ANY MISSING data, with REASONABLE and CLEAR assumptions		

Q1) Draw a block diagram of a generic pattern recognition system and briefly explain the function of each block in your pipeline? [10 Points]

Q2) Consider the data of four adults, indicating their weight and their health status. Devise a simple classifier that can properly classify all four patterns. How a fifth adult of weight 76 kg is classified using this classifier? [5 Points]

None	Yes
50	Unhealthy
60	Healthy
70	Healthy
80	Unhealthy

Q3) Using a naïve Bayes' classifier and the training data in Table 2, classify a given person as male or female based on height, weight, and foot size of 6', 130 lbs, 8", respectively. Assuming no covariance between the features and the data is Gaussian. [Hint Gaussian probability density function with a mean μ and standard deviation

sex	Height (feet)	Weight (lbs)	Foot size (inches)
M	6	180	12
M	5.92 (5' 11")	190	11
M	5.58 (5' 7")	170	12
M	5.92 (5' 11")	165	10
F	5	100	6
F	5.5 (5' 6")	150	8
F	5.42 (5' 6")	130	7
F	5.75 (5' 9")	150	9

σ is given as: $\frac{1}{\sqrt{\sigma^2 2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$] [10 Points].

Q4) Suppose you are given a training sample for 2D data sets as in Table 3. Use the PCA analysis to find the eigenvectors and the percentage of total variance for each principal components. [15 Points]

Feature	Samples									
	1	2	3	4	5	6	7	8	9	10
X ₁	2.5	0.5	2.2	1.9	3.1	2.3	2.0	1.0	1.5	1.2
X ₂	2.4	0.7	2.9	2.2	3.0	2.7	1.6	1.1	1.6	0.9

Q5) Suppose you have a 2D sample space with the samples given in Table 3. Using the k-means clustering algorithm with K=2, find the final class centroids and the total classification error of the k-means clustering. Assume the initial class distribution is C₁ = {1,2,4} and C₂ = {3,5} (Hint: use only two iterations and the Euclidian distance to determine the class membership). [10 Points]

Feature	Samples				
	1	2	3	4	5
F ₁	0	0	1.5	5	5
F ₂	2	0	0	0	2

Best of Luck
Dr. Fahmi Khalifa

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Q1) Draw a block diagram of a generic pattern recognition system and briefly explain the function of each block in your pipeline? [10 Points]

Answer

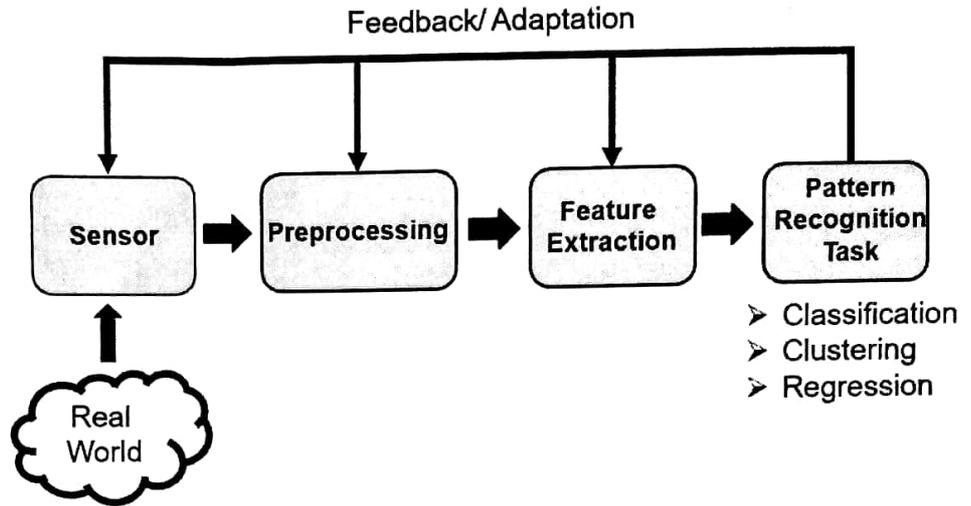
1. The sensing/acquisition stage uses a transducer such as a camera or a microphone to acquire signals (e.g., an image), which should be of sufficient quality that distinguishing "features" can be adequately measured.

2. Pre-processing is often used to remove noise, handle missing data, and detect and handle outlier data.

3. Features extraction is to obtain some attributes that distinguish between classes. The quality of the features is related to their ability to discriminate examples from different classes.

- Examples from the **same class** should have similar feature values, while examples from **different classes** should have different feature values (Discriminating Features)
- Good features should have small **intra-class** variations and large **inter-class** variations

4. The goal of the pattern recognition task is to (i) classify new data (test data) to one of the classes, characterized by a decision region or (ii) to cluster all data samples into a distinct (predetermined) number of classes or (iii) to be used in regression analysis



Q2) Consider the data of four adults, indicating their weight and their health status. Devise a simple classifier that can properly classify all four patterns. How a fifth adult of weight 76 kg is classified using this classifier? [5 Points]

Solution

With this limited data set, the decision boundaries between the two classes should be set at 55 and 65 kg. There is some arbitrariness as to what happens on the actual boundary values, but we could choose for example:

For a weight, x , in kg

If $55 \leq x < 75$ that person is classified as healthy.

The fifth person, at 76kg, should be classified as unhealthy.

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Q3) Using a naïve Bayes' classifier and the training data in Table 2, classify a given person as male or female based on height, weight, and foot size of 6', 130 lbs, 8", respectively. Assuming no covariance between the features and the data is Gaussian. [Hint Gaussian probability density function with a mean μ and standard deviation σ is given as:

$$\frac{1}{\sqrt{\sigma^2 2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2} \quad] \text{ [10 Points].}$$

sex	Height (feet)	Weight (lbs)	Foot size (inches)
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Solution

The means and variances of the features are first found

SEX	Mean (height)	Variance (height)	Mean (weight)	Variance (weight)	Mean (footsize)	Variance (footsize)
Male	5.855	0.035	176.25	0.0122	11.25	0.091
Female	5.417	0.097	132.5	0.0558	7.5	1.666

Assume the prior probabilities are equal, i.e., $P(\text{male}) = P(\text{female}) = 0.5$

For the classification as male the posterior (eqn.4.14 and 4.15) is given by

$$\text{posterior}(\text{male}) = \frac{P(\text{male})P(\text{height} | \text{male}) P(\text{weight} | \text{male})P(\text{footsize} | \text{male})}{\text{evidence}}$$

For the classification as female the posterior is given by

$$\text{posterior}(\text{female}) = \frac{P(\text{female})P(\text{height} | \text{female}) P(\text{weight} | \text{female})P(\text{footsize} | \text{female})}{\text{evidence}}$$

Each of the features are distributed according to a Gaussian, so for height

$$p(\text{height} | \text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(6 - \mu)^2}{2\sigma^2}\right) \approx 1.5789$$

using the appropriate values of μ and σ^2 .

Similarly

$$P(\text{weight} | \text{male}) = 0.00000598; P(\text{footsize} | \text{male}) = 0.0013112$$

$$\text{Using these, the Posterior Numerator (male)} = 0.0000000061984$$

$$P(\text{height} | \text{female}) = 0.223436; P(\text{weight} | \text{female}) = 0.016789; P(\text{footsize} | \text{female}) = 0.28669$$

$$\text{Using these, Posterior Numerator (female)} = 0.00053778$$

Since $\text{posterior}(\text{female}) > \text{posterior}(\text{male})$, then we predict that the sample is female.

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Q4) Suppose you are given a training sample for 2D data sets as in Table 3. Use the PCA analysis to find the eigenvectors and the percentage of total variance for each principal components. [15 Points]

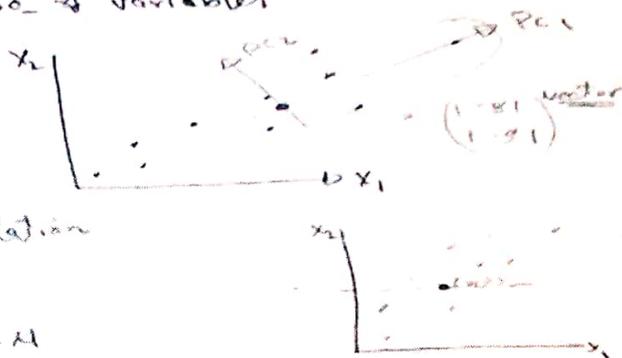
Table 3: Training samples for Q4

Feature	Samples									
	1	2	3	4	5	6	7	8	9	10
X ₁	2.5	0.5	2.2	1.9	3.1	2.3	2.0	1.0	1.5	1.2
X ₂	2.4	0.7	2.9	2.2	3.0	2.7	1.6	1.1	1.6	0.9

x_1 | 2.5 | 0.5 | 2.2 | 1.9 | 3.1 | 2.3 | 2.0 | 1.0 | 1.5 | 1.1
 x_2 | 2.4 | 0.7 | 2.9 | 2.2 | 3.0 | 2.7 | 1.6 | 1.1 | 1.6 | 0.9

$N = 10$ $P = 2$ - no. of variables

→ scatter plot



$r = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$
 $r = 0.926$ positive correlation
 $\mu_1 = 1.81$ $\mu_2 = 1.91$

→ recenter the dataset → X - μ

x_1 | 0.69 | -1.31 | 0.39 | 0.09 | 1.29 | 0.49 | 0.19 | -0.81 | -0.31 | -0.71
 x_2 | 0.49 | -1.21 | 0.99 | 0.19 | 1.09 | 0.79 | -0.31 | -0.81 | -0.31 | -1.01

$X = \begin{bmatrix} 0.69 & 0.49 \\ -1.31 & -1.21 \\ 0.39 & 0.99 \\ \vdots & \vdots \\ -0.71 & -1.01 \end{bmatrix}$ $n \times 2$

$\text{mean } \bar{\mu} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ mean at center point

Covariance matrix $C = \frac{1}{N-1} (X - \bar{\mu})^T (X - \bar{\mu}) = \frac{1}{N-1} X^T X$

$C = \frac{1}{(10-1)} \begin{bmatrix} 2.69 & -1.31 \\ 0.49 & -1.21 \end{bmatrix} = \frac{1}{9} \begin{pmatrix} 2.69 & -1.31 \\ -1.31 & 1.46 \end{pmatrix}$ 2×2

→ find eigenvalues by solve $(C - \lambda I) = 0$

$\det \begin{pmatrix} 0.6166 - \lambda & -0.145 \\ -0.145 & 0.1633 - \lambda \end{pmatrix} = 0$

$(0.6166 - \lambda)(0.1633 - \lambda) - (-0.145)^2 = 0$

Solution

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$\lambda_1 = 1.2840$ $\lambda_2 = 0.0490$
 $\sum W = \lambda W$
 Find eigen vector

$$\begin{bmatrix} 0.6166 & 0.615 \\ 0.615 & 0.7166 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \lambda_1 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

$$\text{vector}_1 = \begin{bmatrix} 0.678 \\ 0.755 \end{bmatrix}, \text{vector}_2 = \begin{bmatrix} 0.735 \\ -0.683 \end{bmatrix}$$

$\text{trace}(Z) = \text{sum of eigen values}$

$$\% \text{ of total variance} = \frac{\lambda}{\text{total}}$$

$$\text{total} = \sum \lambda_{ii}$$

we can discard less significant components

Q6) Suppose you have a 2D sample space with the samples given in Table 3. Using the k-means clustering algorithm with K=2, find the final class centroids and the total classification error of the K-means clustering. Assume the initial class distribution is $C_1 = \{1,2,4\}$ and $C_2 = \{3,5\}$ (Hint: use only two iterations). [10 Points]

Solution

	Samples				
Feature	1	2	3	4	5
F ₁	0	0	1.5	5	5
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2. Random distribution of samples:

$$C_1 = \{x_1, x_2, x_4\} \text{ and } C_2 = \{x_3, x_5\}$$

3. Centroids:

$$M_1 = \{(0+0+5)/3, (2+0+0)/3\} = \{1.66, 0.66\}$$

$$M_2 = \{(1.5+5)/2, (0+2)/2\} = \{3.25, 1.00\}$$

Within-cluster variations:

$$e_1^2 = [(0-1.66)^2 + (2-0.66)^2] + [(0-1.66)^2 + (0-0.66)^2] + [(5-1.66)^2 + (0-0.66)^2] = 19.36$$

$$e_2^2 = [(1.5-3.25)^2 + (0-1)^2] + [(5-3.25)^2 + (2-1)^2] = 8.12$$

Total square error:

$$E^2 = e_1^2 + e_2^2 = 19.36 + 8.12 = 27.48$$

2. Reassign all samples:

$d(M_1, x_1) = 2.14$	and	$d(M_2, x_1) = 3.40$	\Rightarrow	$x_1 \in C_1$
$d(M_1, x_2) = 1.79$	and	$d(M_2, x_2) = 3.40$	\Rightarrow	$x_2 \in C_1$
$d(M_1, x_3) = 0.83$	and	$d(M_2, x_3) = 2.01$	\Rightarrow	$x_3 \in C_1$
$d(M_1, x_4) = 3.41$	and	$d(M_2, x_4) = 2.01$	\Rightarrow	$x_4 \in C_2$
$d(M_1, x_5) = 3.60$	and	$d(M_2, x_5) = 2.01$	\Rightarrow	$x_5 \in C_2$

3. New clusters:

$$C_1 = \{x_1, x_2, x_3\} \text{ and } C_2 = \{x_4, x_5\}$$

New centroids:

$$M_1 = \{0.5, 0.67\}$$

$$M_2 = \{5.0, 1.0\}$$

Errors:

$$e_1^2 = 4.17, \quad e_2^2 = 2.00 \quad \rightarrow \quad E^2 = 6.17$$