Mansoura University
Faculty of Engineering

Final Exam.
Thursday 26/5/2016

Biomedical Engineering Programs

Prof. Dr. Magdi S. El-Azab | Mathematics 4 (Math 102) | Time allowed: 2 hrs.

Solve the following questions (Full mark 50 pts.)

1. (a) [5 pts.] Evaluate the double integral $\int_0^4 \int_{\sqrt{y}}^2 9\sqrt{x^3+1}\, dxdy$.

(b) [5 pts.] Find the volume of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y + 5$.

(c) [10 pts.] Verify Stokes' theorem for the vector field

$$\vec{F}(x,y,z) = 2y\,\mathbf{i} - 5z\,\mathbf{j} - 3x\,\mathbf{k},$$

and S is the solid bounded by the paraboloid $z = 9 - (x^2 + y^2)$, $z \geq 0$.

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2. (a) [5 pts.] Using the graph in the xy-plane, classify the following partial differential equation

$$xu_{xx} + 2u_{xy} + xu_{yy} + 4u_y - 6xy = 0$$

(b) [5 pts.] Build a model that describes the temperature distribution in a rod of length π made of homogeneous metal with constant cross section A that is completely insulated along its lateral edges.

(c) [10 pts.] Use the technique of separation of variables to solve the following boundary value problem:

$$u_t = 4u_{xx}, \qquad 0 \leq x \leq \pi, \quad t \geq 0,$$
$$u_x(0,t) = u_x(\pi,t) = 0, \qquad t \geq 0$$
$$u(x,\pi) = 1 - \frac{x}{\pi}, \qquad 0 \leq x \leq \pi$$

3. (a) [5 pts.] Find Fourier integrals of the function

$$f(x) = \begin{cases} \sin x, & -\pi \le x \le \pi \\ 0, & otherwise \end{cases}$$

(b) [5 pts.] Use the divergence theorem to evaluate $\iint_S \vec{F} \cdot dS$, where

$$\vec{F}(x, y, z) = (5x - y\cos y)\,\mathbf{i} + (2y + 4\sin z)\mathbf{j} + (3z + 9e^x)\mathbf{k}$$

and S is the surface of the region R bounded by the solid that lies above the cone $z = \sqrt{3(x^2 + y^2)}$ and below the sphere $x^2 + y^2 + z^2 = 4z$.

(c) [2 pts.] Evaluate the line integral $\int_C \left(x\sqrt{y}\,dx + y\sqrt{x}\,dy\right)$ where C consists of the arc of the circle $x^2 + y^2 = 1$ from $(1,\ 0)$ to $(0,\ 1)$ and the line segment from $(0,\ 1)$ to $(0,\ 2)$.