Exam Guidelines: This Exam contains **5** questions in **2** pages. Start every question in a new page.

Question (1) [10 pt.]

a) [5 pt.] A box **I** contains 9 light bulbs of which 4 are defective, box **II** contains 9 light bulbs of which 5 are defective, and box **III** contains 9 light bulbs of which 3 are defective. We select a box at random and then select two bulbs at random from it. Find the probability that:

i) the two bulbs are defective,

ii) the two bulbs are selected from the urn **III** given that the two bulbs are defective.

b) [5 pt.] Prove that: if three events A, B and C are independent events, then $(A \cup B)$ and C are independent.

Question (2) [10 pt.]

a) [5 pt.] 10 students enter an exam of 50 *degree full mark*. Find the probability that:

i) they got different degrees,

ii) at least three students got the same degree.

b) [5 pt.] If the number of misprints, denoted by X, on a single page of a book has a Poisson distribution such that $P(X=1)=P(X=2)$. **i)** What is the probability that there is at least two errors in this page, **ii)** Find also, the probability that there is 5 errors in a given three pages.

Question (3) [10 pt.]

a) [5 pt.] When we request a call from base station, the call may be blocked with probability 0.1, terminated with probability 0.3, or make successfully with probability 0.6. If we request 10 calls, what is the probability to get:

i) 2 blocked and 5 success calls,

ii) 6 success calls.

iii) If call request continued until it make successfully, what is the probability that the first success call occurs at the 5^{th} request.

b) [5 pt.] If the probability density function of a continuous random variable is given by

$$f(x)=\begin{cases} e^{2x} & x\le 0 \\ kx & 0<x\le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find: **i)** the value of k, **(ii)** the mean and variance, **iii)** $P(-0.5\le x\le 0.5)$.

Question (4) [10 pt.]

a) [5 pt.] The time to failure of fans in a personal computer can be modeled by an exponential distribution with mean 4000 hours. Find the probability that the fans will fail:

(i) after 8000 hours, **(ii)** between 5000 and 10000 hours.

iii) Determine a such that the probability that the fans will fail before a hours is 0.75.

b) [5 pt.] The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. If it found that this follow normal distribution with standard deviation $\sigma=15.7$. A sample of 25 tubes results in $\bar{x}=317.2$ and $s=15.5$. **i)** Find in a 95 % confidence interval for the mean of the current required. **ii)** Find the number of tubes, n, taken as a sample to reduce the confidence interval length to the halfe of it in part **(i)**.

Question (5) [10 pt.]

a) [5 pt.] Consider the sugar content of the syrup in canned peaches. Suppose that the variance is thought to be $\sigma^2=18$ (milligrams)^2. A random sample of size $n = 9$ cans yields a sample standard deviation of $S=4.8$ milligrams. Using 5 % level of significance; Test the hypothesis: H_0: $\sigma^2=18$ versus H_1: $\sigma^2\neq 18$.

b) [5 pt.] For the following data of the two variables X and Y; find:

i) The equation of linear regression of Y on X.

ii) The coefficient linear correlation between X and Y.

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

With all best wishes
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