



**Attempt All Questions. Assume Any Missing Data. Questions Are Equal Weight But Are Not Equal Difficulty.**

**Question (1)**

- Mention the names of three conservation laws.
- Mention the names of two forces in the definition of Reynolds number (Re).
- Mention the names of two different methods used to solve the differential equations.
- What is the difference between Laplace equation & Poisson equation? What is the type of these partial differential equations (PDEs)? Why?

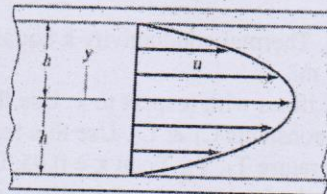
**Question (2)**

The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates as shown in Figure 1 can be modeled using the equation

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial P}{\partial x}$$

The maximum velocity occurs at centerline ( $y = 0$ ). The bottom ( $y = -h$ ) and top ( $y = +h$ ) walls of the plate are kept at zero velocity.

- Integrate the above equation 2 times with respect to  $y$ . Use the 2 boundary conditions at centerline and at the bottom wall to obtain the values of the  $C_1$  &  $C_2$ .
- Use this velocity distribution equation to obtain the expression of the maximum velocity.
- Determine the volumetric flow rate ( $Q$ ). Take the plate width =  $b$ .
- Determine the mean velocity ( $V$ ).
- Obtain the velocity distribution equation as a function of the following parameters: the mean velocity ( $V$ ), the half plate height ( $h$ ), and the  $y$ -axis.



**Question (3)**

For the following velocity distribution.

$$u/U_\infty = a + b\eta + c\eta^2$$

Where  $a$ ,  $b$ ,  $c$  are constants and  $\eta = y/\delta$

- Write the three boundary conditions required to determine the constants  $a$ ,  $b$ ,  $c$ .
- By solving these three equations, obtain the values of constants  $a$ ,  $b$ ,  $c$ .
- Calculate the displacement thickness and momentum thickness in terms of  $\delta$ .



**Question (4)**

Consider the steady-state temperature distribution for a long, thin rod positioned between two constant-temperature walls. The rod's cross sectional dimensions are small enough so that radial temperature gradients are minimal and, consequently, temperature is a function exclusively of the axial coordinate  $x$ . Heat is transferred along the rod's longitudinal axis by conduction and between the rod and the surrounding gas by convection. Radiation is assumed to be negligible. Using a heat balance for a differential element of a heated rod subject to conduction and convection, the differential equation can be expressed as

$$\frac{d^2T}{dx^2} + h'(T_\infty - T) = 0$$

where  $h' =$  a bulk heat-transfer parameter reflecting the relative impacts of convection and conduction  $= 2h/(rk)$ .

- i. Obtain the units of  $h'$  in 2 different ways.
- ii. Show that

$$\frac{d^2T}{dx^2} \equiv \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2}$$

- iii. Use the finite-difference approach to get the  $4 \times 4$  matrix for a 10-m rod with  $h' = 0.01$ ,  $T_\infty = 20^\circ\text{C}$ , and the boundary conditions:  $T(0) = 40^\circ\text{C}$ ,  $T(10) = 200^\circ\text{C}$ . Use four interior nodes with a segment length of  $\Delta x = 2$  m.

**Question (5)**

Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of  $T_0 = 100^\circ\text{C}$  and  $T_4 = 340^\circ\text{C}$  respectively. The one-dimensional problem is governed

$$\frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0$$

Take rod length  $L$  equals 0.3 m, Thermal conductivity  $k$  equals 1000 W/m.K, cross-sectional area  $A$  is  $10 \times 10^{-3} \text{ m}^2$ ,  $\delta x = 0.1$  m.

- i. Integrate the above equation 2 times with respect to  $x$ . Use the 2 boundary conditions at rod ends to obtain the values of the constants  $C_1$  &  $C_2$ . Use this temperature distribution equation to obtain the values of the temperature  $T_1, T_2, T_3$ , at  $x = 0.05, 0.15, 0.25$  m respectively.
- ii. Using the control volume method, obtain the discretised equation for nodal point 2, the discretised equation for nodal point 1, and the discretised equation for nodal point 3.

Good Luck  
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