Mansoura University
Faculty of Engineering

Final 1st 2017/2018
BME Program
Level 000
Exam Date: 26-12- 2017

Course Name: Math 1
Course Code:MATH001
Total marks: 50
Allowed Time: 2 Hours

Answer the following questions

Question 1 [6, 5, 4 marks] /15

a) Test the following function for maximum and minimum $z = x^3 - 3x^2 + \cosh y$

Solution	$z_{xx} =$
$z_x =$	$z_{yy} =$
$z_y =$	$z_{xy} =$
Critical Points	$D =$
∴ the critical points are (,), and (,)	

Test the critical points

(,)	(,)
∴ (,) is point	∴ (,) is point

$f(x)$	e^x	a^x	$\tan x$	$\ln x$	$\sin x$	$\cosh x$	$\sinh x$	$\sec^{-1} x$	$\cosh^{-1} x$
$f'(x)$	e^x	$a^x \ln a$	$\sec^2 x$	$\frac{1}{x}$	$\cos x$	$\sinh x$	$\cosh x$	$\frac{1}{x\sqrt{x^2-1}}$	$\frac{1}{\sqrt{x^2-1}}$

b) Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial w}$ for the function: $z = \sec^{-1}(e^x) + x \tan y$,

$x = u^3 + \cosh^{-1} w$, , and $y = 1 + w^2$

Solution

The Tree diagram

$\frac{\partial z}{\partial u} =$	$\frac{\partial z}{\partial w} =$

c) If $z = f(x, y)$ is defined by the relation $\sin(xy) + z^y + \ln(x^2 + z) = 0$, find $\frac{\partial z}{\partial y}$

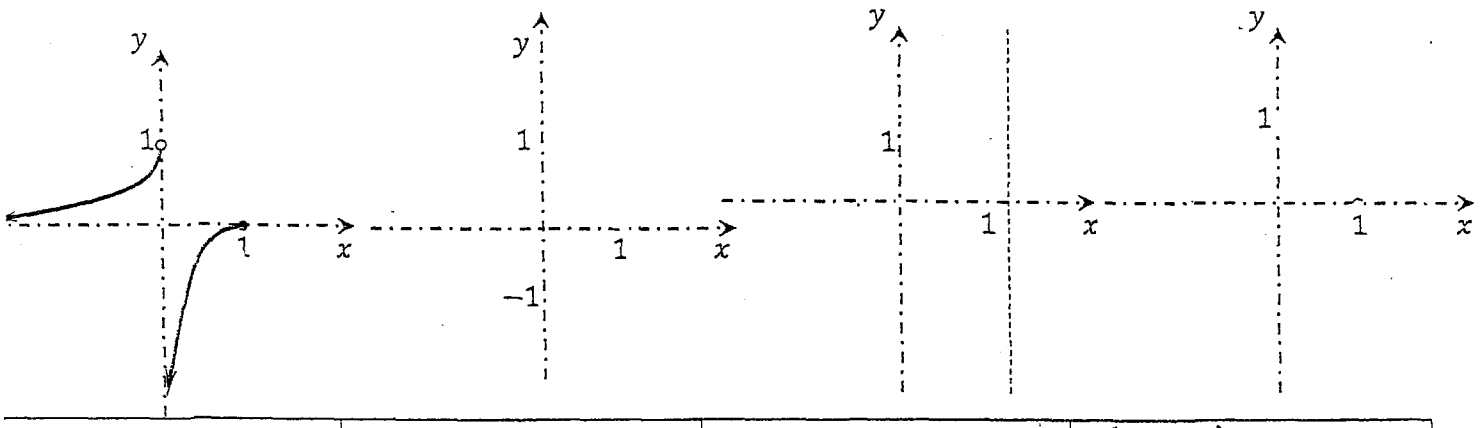
Solution

Question 2 [5, 5 marks]

/10

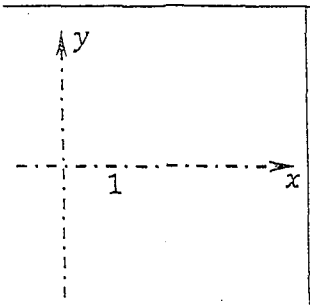
a) Use the given function to complete the following figures and the table

$f(x)$ $|f(x)|$ $1/f(x)$ $f^{-1}(x)$

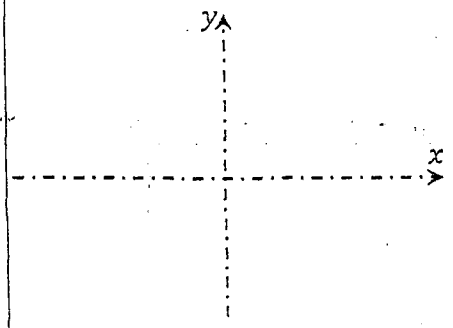


Range $f(x) =$	$\lim_{x\to-\infty}(f(x)) =$	$\lim_{x\to0^+}(f(x)) =$	$f(f^{-1}(x)) = x$ if $x \in$

b) Find the domain and the range of the function $z = \ln(y - \tanh x)$



Solution



Domain z

Question 3 [4, 3, 3 marks]

/10

a) Evaluate the limit $\lim_{x \to 0^+} (1 + \sinh x)^{\frac{1}{\sinh x}}$

Solution

b) Complete the following table:

Final augmented matrix form	The Solution of the system (if it exist)
$\left(\begin{array}{ccc\|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$	$x =$ $y =$ $z =$
$\left(\begin{array}{ccc\|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$	$x =$ $y =$ $z =$
$\left(\begin{array}{ccc\|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$	$x =$ $y =$ $z =$

c) If $AC = CA = I$, *where* $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 2 & 1 \end{pmatrix}$ *and* $C = \begin{pmatrix} 4 & -3 & 1 \\ 1 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix}$

Then, complete the following

i) $C^{-1} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$

ii) The solution of the system $\begin{aligned} x + y + 2z &= 1 \\ x + 2y + 3z &= 1 \\ 2y + z &= 1 \end{aligned}$

is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$

Question 4 [5, 5, 5 marks] /15

a) If $x = 0$ is a root of multiplicity two of $D(x) = x^4 + x^2 + ax + b$

i) Find a *and* b (using synthetic division)

ii) True or False: $x^2 + 1$ is a factor of $D(x)$

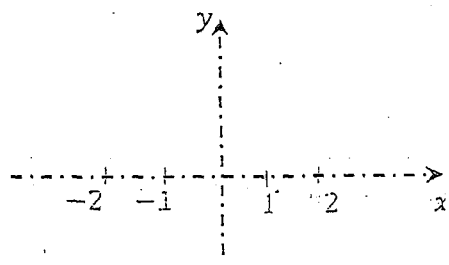
iii) Sketch the graph of $D(x)$

Solution: i) Synthetic division

$\therefore a =$, $b =$

ii) True or False: $x^2 + 1$ is a factor of $D(x)$

iii) The graph of $D(x)$ is



b) Use partial fractions to decompose $\frac{x^3-x^2-1}{x^4+x^2}$