Subject: Math 2
Course Code: MTH003
Level: 000
Allowed Hours: 2Hours
27/5/2018
Final 2nd 2017/2018

Biomedical Engineering

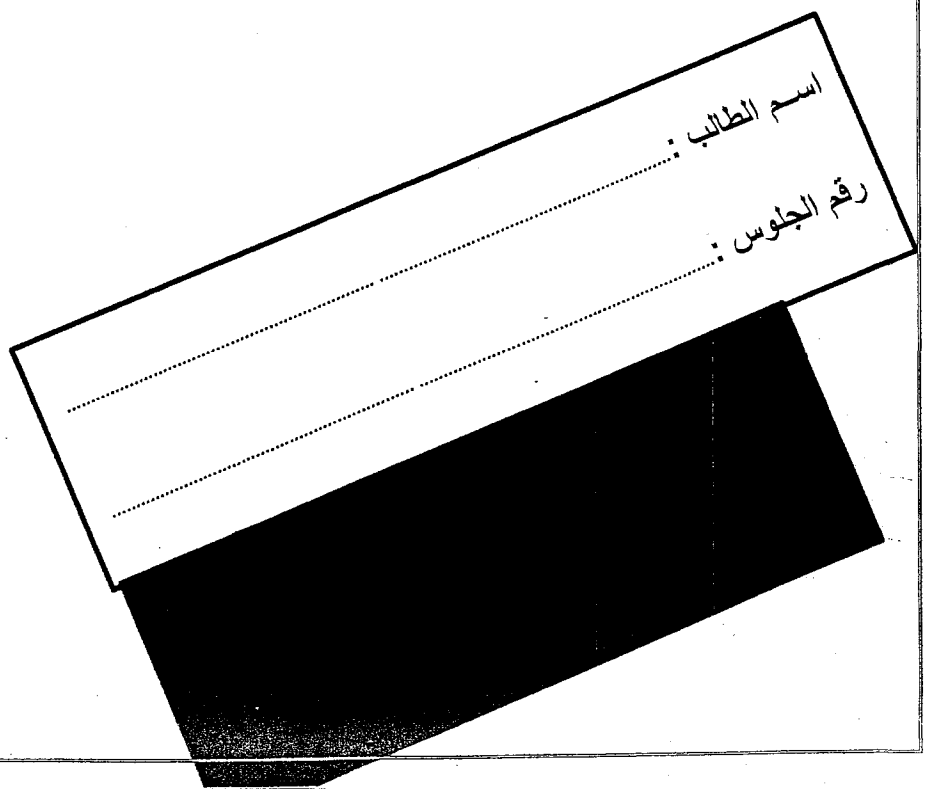
Mansoura University
Faculty Of Engineering

**Fa\Sp\Sum Semester Exam .....Year 2018**

Course:.....Date: / / 2018

Year FR SO JR SR1 SR2

| Question No. | Mark | Examiner Signature |
|---|---|---|
| One | | |
| Two | | |
| Three | | |
| Four | | |
| Five | | |
| **Total Marks (digits)** | | |
| **Total Marks (letters)** | | |
| **Signature** | | |



## BASIC RULES

| | | |
|---|---|---|
| $\int \frac{dx}{1+x^2}$ | $\int \frac{dx}{1-x^2}$ | $\int \frac{dx}{x^2-1}$ |
| $\tan^{-1} x$ | $\tanh^{-1} x$ | $-\coth^{-1} x$ |
| $\int \frac{dx}{\sqrt{1-x^2}}$ | $\int \frac{dx}{\sqrt{1+x^2}}$ | $\int \frac{dx}{\sqrt{x^2-1}}$ |
| $\sin^{-1} x$ | $\sinh^{-1} x$ | $\cosh^{-1} x$ |
| $\int \frac{dx}{x\sqrt{x^2-1}}$ | $\int \frac{dx}{x\sqrt{1-x^2}}$ | $\int \frac{dx}{x\sqrt{1+x^2}}$ |
| $\sec^{-1} x$ | $-\operatorname{sech}^{-1} x$ | $-\operatorname{csch}^{-1} x$ |

| | |
|---|---|
| $\int \sinh x\, dx$ | $\cosh x$ |
| $\int \cosh x\, dx$ | $\sinh x$ |
| $\int \operatorname{sech}^2 x\, dx$ | $\tanh x$ |
| $\int \operatorname{csch}^2 x\, dx$ | $-\coth x$ |
| $\int \operatorname{sech} x \tanh x\, dx$ | $-\operatorname{sech} x$ |
| $\int \operatorname{csch} x \coth x\, dx$ | $-\operatorname{csch} x$ |

| | |
|---|---|
| $\int \sin x\, dx$ | $-\cos x$ |
| $\int \cos x\, dx$ | $\sin x$ |
| $\int \sec^2 x\, dx$ | $\tan x$ |
| $\int \csc^2 x\, dx$ | $-\cot x$ |
| $\int \sec x \tan x\, dx$ | $\sec x$ |
| $\int \csc x \cot x\, dx$ | $-\csc x$ |

| | | | |
|---|---|---|---|
| $\int e^x\, dx$ | $e^x$ | $\int x^n\, dx$ | $x^{n+1}/(n+1)$ |
| $\int a^x\, dx$ | $a^x/\ln a$ | $\int dx$ | $x$ |
| $\int \frac{1}{x}\, dx$ | $\ln x$ | $\int \frac{1}{\sqrt{x}}\, dx$ | $2\sqrt{x}$ |

$$L = \int dL, \qquad S = 2\pi \int r\, dL,$$

$$dL = \sqrt{1+(dy/dx)^2}\ dx = \sqrt{1+(dx/dy)^2}\ dy = \sqrt{(dx/dt)^2+(dy/dt)^2}\ dt$$

$$A = \int_a^b (y_{out} - y_{in})\ dx = \int_c^d (x_{out} - x_{in})\ dy$$

$$V = \pi \int_a^b ((y_{out})^2 - (y_{in})^2)\ dx = \pi \int_c^d ((x_{out})^2 - (x_{in})^2)\ dy$$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\ dt = f(b(x))\ \ b'(x) - f(a(x))\ \ a'(x)$$

| | |
|---|---|
| $(\ )^2 = (\ )$ | parabola |
| | |
| $(\ )^2 + (\ )^2 =$ | $+$ ellipse |
| | 0 point |
| | $-$ nothing |
| | |
| $(\ )^2 - (\ )^2 =$ | $\pm$ hyperbola |
| | 0 pair of lines |

| | $x =$ | $y =$ | $z =$ |
|---|---|---|---|
| $x-axis$ | $t$ | 0 | 0 |
| $y-axis$ | 0 | $t$ | 0 |
| $z-axis$ | 0 | 0 | $t$ |

| $xy-plane$ | $xz-plane$ | $yz-plane$ |
|---|---|---|
| $z = 0$ | $y = 0$ | $x = 0$ |

$\cos^2 x + \sin^2 x = 1,$

$\cosh^2 x - \sinh^2 x = 1$

$\tan^2 x = \sec^2 x - 1,$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\cosh x + \sinh x = e^x,$

$\cosh x - \sinh x = e^{-x}$

Mansoura University
Faculty of Engineering

Spring Semester Exam.
BME Program
Level 000

Course Name: Math 2
Course Code: MATH003
Exam Date: 27-5- 2018
Allowed Time: 2 Hours

■ أجب كل سؤال في المكان المخصص له ■ إذا لم يكفى المكان المخصص للإجابة يمكن التكملة في الصفحة (6/6) ■ حل بالقلم الرصاص وصفر خطك

## Solve the following problems

### Problem 1 /10

(A) *Find* $\int_{-\frac{\pi}{2}}^{\pi} \left( \frac{2x\, e^{x^2}}{1+\cos x+|\cos x|} \right) dx$ — 5

Solution

(B) The equation $S = 2\pi \int_0^1 \sinh^{-1} t \sqrt{\left(1-\frac{1}{1+t^2}\right)+\left(\frac{dy}{dt}\right)^2}\, dt,$ — 5

represents the surface area generated by revolving a parametric arc ( $x = g(t), y = h(t),\ 0 \le t \le 1$) about the $x-$ axis. Complete the following:

i) $y =$

ii) The length of this arc is $L =$

iii) The area of the surface $S =$

## Problem 2 /10

(A) Solve the initial value problem $(\tan^{-1} y)\, e^{-x}\, dy = (1+y^2)\, \cos(e^x - 1)\, dx, \quad y(0) = 0$

5

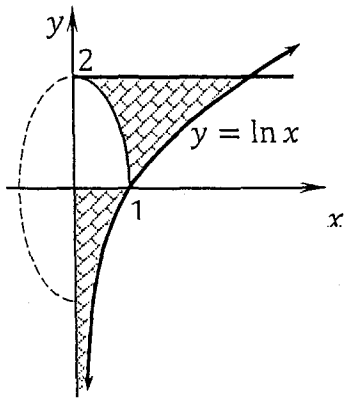
Solution

---

(B) i) For the given figure, the equation of the ellipse is

5

ii) Find the volume of the solid generated by revolving the shaded area about the $y-axis$



**Problem 3** /10

*A) Find* $\int_0^1 \left(\frac{x \sin^{-1} x}{\sqrt{1-x^2}}\right) dx$

/4

Solution

**(A)** For the parabola $x^2 - 4x + 4y = 0$

/6

i) Write the equation of the tangent to the parabola at the origin point *O*.

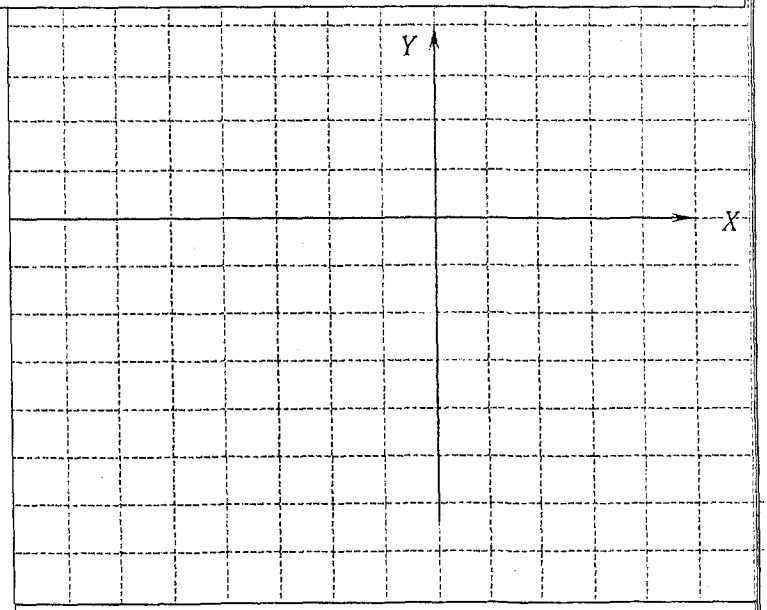
ii) ) Sketch the parabola

iii) Write the equation of a circle which passes through the points *VFL* (Vertex, Focus, end of L.R. of the parabola)

Solution

i)

ii)



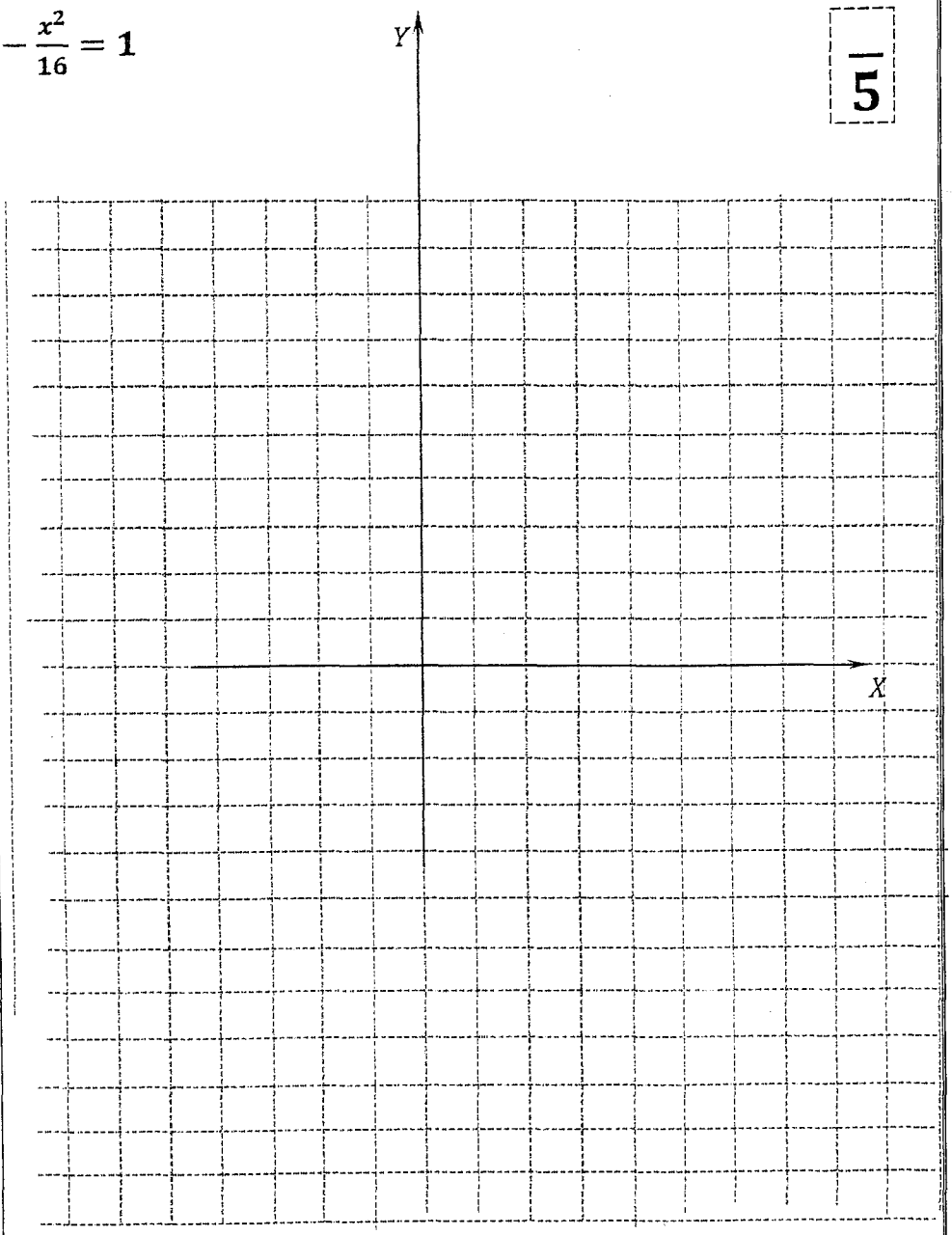

iii)

## Problem 4 /10

(A) Sketch the hyperbola $\frac{(y+5)^2}{9} - \frac{x^2}{16} = 1$

5

Solution




$a^2 =$ $b^2 =$

$e = \sqrt{\phantom{xxxx}} =$

$b =$

$be =$

$\frac{b}{e} =$

$\frac{a^2}{b} =$

(B) Find the value of $c$ which makes the angle between the line $L$: $x = 1 + 2t$, $y = t$, $z = -1 + t$ and the plane $\pi$: $x - y + c\,z = 2$ equal 30°

3

Solution

C) For the $x$-ellipse $\frac{x^2}{9} + \frac{y^2}{9} = 1$,

2

| center | eccentricity | Foci | vertices |
|---|---|---|---|
| ( , ) | $e =$ | $F = ($ , $)$, $F' = ($ , $)$ | ( , ) |

## Problem 5 /10

**(A)** For the surface $\frac{(x+1)^2}{4}+\frac{y^2}{9}+\frac{(z-2)^2}{4}=1$ complete the following

/4

a) The name of the surface is ..... (paraboloid, ellipsoid, hyperboloid, cone)

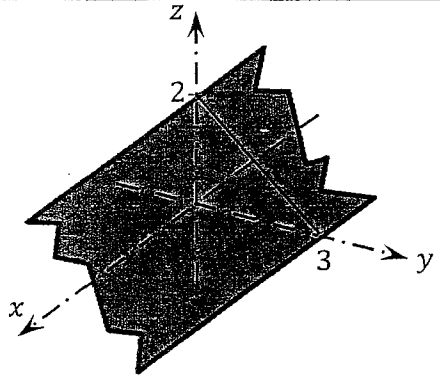
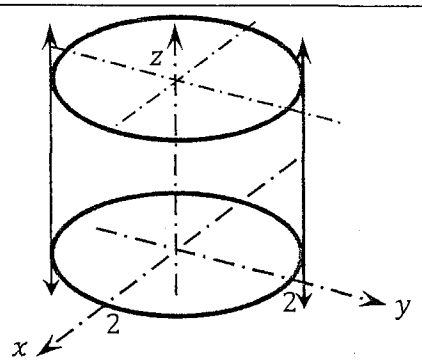
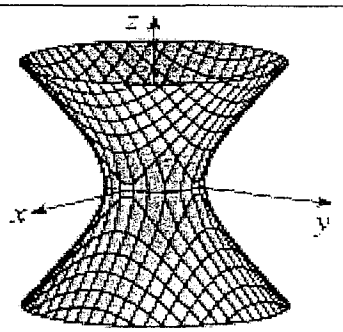
b) The center of the surface is ( , , )

c) The vertices of the surface are ( , , )

d) The parametric equation of its axis is $x=$ , $y=$ , $z=$

**(B)** Write a suitable equation for each of the following:

/3

| | | |
|---|---|---|
|   |   |   |
| The equation of the plane is | The equation of the cylinder is | The equation of the hyperboloid is |

**(C)** For the surface $(x+1)^2-y^2=-4z$

/3

a) The name of the surface is

i) hyperboloid 1sheet ii) hyperboloid 2 sheets iii) elliptic paraboloid iv) hyperbolic paraboloid

b) The saddle point is

i) $(-1,0,0)$ ii) $(0,0,0)$ iii) $(1,-1,-4)$ iv) $(-1,0,-1)$

c) The intersection of the surface with the $x=-1$ has vertex

i) $(0,0,0)$ ii) $(-1,0,0)$ iii) $(1,-1,-4)$ iv) $(0,0)$

d) The intersection of the surface with the $xz-plane$ has eccentricity

i) $e=0$ ii) $e=1/2$ iii) $e=1$ iv) $e=\sqrt{2}$

f) The intersection of the surface with the $z=-1$ has eccentricity

i) $e=0$ ii) $e=1/2$ iii) $e=1$ iv) $e=\sqrt{2}$

g) The point(s) of intersection of the surface with the $y-axis$ is(are)

i) $(0,0,0)$ ii) $(-1,0,0)$ iii) $(0,0,-1/4)$ iv) $(0,\pm1,0)$

| | |
|---|---|
| a | |
| b | |
| c | |
| d | |
| f | |
| g | |

My best wishes to all of you! Reda Abdou