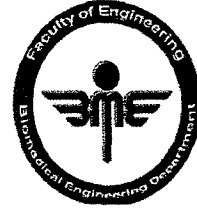




Mansoura University  
Faculty of Engineering

Final Examination  
Saturday 12/5/2018



Biomedical Engineering Programs

Prof. Dr Magdi S. El-Azab Sawwan

Mathematics 4 (MTH 102)

Time allowed: 2 Hours.

**Solve the following questions. Assume any missed data. (Full mark 50 pts.)**

**Question 1.**

(a) [5 pts.] Solve in series of the following differential equation near the point  $x_0 = 0$ :

$$2xy'' + y' + x^3y = 0$$

(b) [5 pts.] Find the Fourier sine series of the function  $f(x) = x(\pi - x)$ ,  $0 \leq x \leq \pi$ . Sketch the graph of the function in the range  $-4\pi \leq x \leq 4\pi$ .

(c) [15 pts.] Prove that:

(i)  $\Gamma(x + 1) = x\Gamma(x)$ , and hence find  $\frac{\Gamma(3)\Gamma(3/2)}{\Gamma(9/2)}$  in its simplest form.

(ii)  $\int_1^2 2(x-1)^{n-1}(2-x)^n dx = \frac{(\Gamma(n))^2}{\Gamma(2n)},$

(iii) the system  $\{e^{imx}, e^{inx}\}$ , is orthogonal over the interval  $[0, 2\pi]$ , for all positive integers  $m$  and  $n$ , such that  $m \neq n$ .

**Question 2.**

(a) [5 pts.] Find the Fourier sine integral representation of the function

$$f(x) = e^{-2x} \cos x, \quad x > 0$$

(b) [5 pts.] Evaluate each of the following integrals

(i)  $I_1 = \int_1^\infty e^{2x-x^2} dx,$       (ii)  $I_2 = \int_0^{\pi/2} \tan^5 \theta \cos^7 \theta d\theta$

(c) [5 pts.] Using the graph in the  $xy$ -plane, classify the following partial differential equation

$$(y - 2)u_{xx} + 2(x - 1)u_{xy} - yu_{yy} = 0$$

(d) [10 pts.] By the separation of variables technique solve the following boundary value problem:

$$u_{tt} = 4u_{xx}, \quad 0 \leq x \leq \pi, \quad t > 0,$$

$$u(0, t) = u(\pi, t) = 0, \quad t > 0$$

$$u(x, 0) = u_t(x, 0) = 2x, \quad 0 \leq x \leq \pi$$

---

With all best wishes