Mansoura University
Faculty of Engineering

Final Examination
Saturday 12/5/2018

Biomedical Engineering Programs

Prof. Dr Magdi S. El-Azab Sawwan | **Mathematics 4 (MTH 102)** | **Time allowed: 2 Hours.**

Solve the following questions. Assume any missed data. (Full mark 50 pts.)

Question 1.

(a) [5 pts.] Solve in series of the following differential equation near the point $x_0 = 0$:

$$2xy'' + y' + x^3y = 0$$

(b) [5 pts.] Find the Fourier sine series of the function $f(x) = x(\pi - x), 0 \le x \le \pi$. Sketch the graph of the function in the range $-4\pi \le x \le 4\pi$.

(c) [15 pts.] Prove that:

(i) $\Gamma(x+1) = x\Gamma(x)$, and hence find $\dfrac{\Gamma(3)\Gamma(3/2)}{\Gamma(9/2)}$ in its simplest form.

(ii) $\displaystyle\int_1^2 2(x-1)^{n-1}(2-x)^n\,dx = \frac{\left(\Gamma(n)\right)^2}{\Gamma(2n)}$,

(iii) the system $\{e^{imx},\ e^{inx}\}$, is orthogonal over the interval $[0, 2\pi]$, for all positive integers m and n, such that $m \neq n$.

Question 2.

(a) [5 pts.] Find the Fourier sine integral representation of the function

$$f(x) = e^{-2x}\cos x,\ \ x > 0$$

(b) [5 pts.] Evaluate each of the following integrals

(i) $I_1 = \displaystyle\int_1^\infty e^{2x-x^2}\,dx$, ($ii$) $I_2 = \displaystyle\int_0^{\pi/2} \tan^5\theta\cos^7\theta\,d\theta$

(c) [5 pts.] Using the graph in the xy-plane, classify the following partial differential equation

$$(y-2)u_{xx} + 2(x-1)u_{xy} - yu_{yy} = 0$$

(d) [10 pts.] By the separation of variables technique solve the following boundary value problem:

$$\begin{aligned} &u_{tt} = 4u_{xx}, && 0 \le x \le \pi, \quad t > 0, \\ &u(0,t) = u(\pi,t) = 0, && t > 0 \\ &u(x,0) = u_t(x,0) = 2x, && 0 \le x \le \pi \end{aligned}$$

With all best wishes