Mansoura University Faculty of Engineering Dept. of Power Mech. Eng. Course Title: 3D Modeling & Simulation of Heat Transfer & Fluid Flow Course Code: MPE371		BME Level 300 May 2018 Exam Type: Final Time: 2 Hours Full Mark: 50

Attempt All Questions. Assume Any Missing Data. Questions Are Equal Weight But Are Not Equal Difficulty.

Question (1)

1.a Define the momentum.

1.b Define the Prandtl number (Pr). Give examples for the fluids with low, and high Prandtl number.

1.c Write the Fourier's law of heat conduction. Why do we put the negative sign in the Fourier's law of heat conduction?. When does the heat flux (q) equal zero?

1.d Mention the names of three types of partial differential equations (PDEs). Explain how can we classify the partial differential equations (PDEs) into three categories.

Question (2)

Two infinite, parallel plates separated by a distance h. One plate, the bottom one, translates with a constant velocity U in its own plane while the upper plate is kept fixed. Neglecting pressure gradients, the Navier–Stokes equations simplify to

$$\frac{d^2u}{dy^2} = 0$$

where y is a spatial coordinate normal to the plates and u (y) is the velocity distribution. Determine.

i. Integrate the above equation 2 times with respect to y. Write the two boundary conditions required to determine the constants C_1, C_2.

ii. By solving these two equations, obtain the values of constants C_1, C_2.

iii. Determine the volumetric flow rate (Q). Take the plate width = b.

iv. Determine the mean velocity (V).

v. Obtain the velocity distribution equation as a function of the following parameters: the mean velocity (V), the plate height (h), and the y-axis.

Question (3)

For the following velocity distribution.

$$u/U_\infty = a + b\eta + c\eta^2 + d\eta^3$$

Where a, b, c, d are constants and $\eta = y/\delta$

i. Write the four boundary conditions required to determine the constants a, b, c, d.

ii. By solving these four equations, obtain the values of constants a, b, c, d.

iii. Calculate the displacement thickness (δ^*) and momentum thickness (θ) in terms of δ.

Question (4)

Consider the steady-state temperature distribution for a long, thin rod positioned between two constant-temperature walls. The rod's cross sectional dimensions are small enough so that radial temperature gradients are minimal and, consequently, temperature is a function exclusively of the axial coordinate x. Heat is transferred along the rod's longitudinal axis by conduction and between the rod and the surrounding gas by convection. Radiation is assumed to be negligible. Using a heat balance for a differential element of a heated rod subject to conduction and convection, the differential equation can be expressed as

$$\frac{d^2T}{dx^2} + h'(T_\infty - T) = 0$$

where h' = a bulk heat-transfer parameter reflecting the relative impacts of convection and conduction = 2h/(rk).

i. Obtain the units of h' in 2 different ways.

ii. Show that

$$\frac{d^2T}{dx^2} \cong \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2}$$

iii. Use the finite-difference approach to get the 4 x 4 matrix for a 10-m rod with h' = 0.04, T_∞ = 20°C, and the boundary conditions: T (0) = 40°C, T (10) = 200°C. Use four interior nodes with a segment length of Δx = 2 m.

Question (5)

Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of T_O = 120°C and T_5 = 440°C respectively. The one-dimensional problem is governed

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

Take rod length L equals 0.4 m, Thermal conductivity k equals 1000 W/m.K, cross-sectional area A is 10×10^{-3} m^2, δx = 0.1 m.

i. Integrate the above equation 2 times with respect to x. Use the 2 boundary conditions at rod ends to obtain the values of the constants C_1 & C_2. Use this temperature distribution equation to obtain the values of the temperature T_1, T_2, T_3, T_4 at x = 0.05, 0.15, 0.25, 0.35 m respectively.

ii. Using the control volume method, obtain the discretised equation for nodal points 2 and 3, the discretised equation for nodal point 1, and the discretised equation for nodal point 4.

Good Luck

Dr. M. M. Awad