(1) Extended Surface

- **Temperature distribution and heat transfer from fin of uniform cross section**

Tip Condition (at x=L)	Temperature distribution: θ/θ_b	Fin heat transfer rate, q_f
Convection heat transfer: $h\,\theta(L) = -k\left(\frac{d\theta}{dx}\right)_{x=L}$	$\dfrac{\cosh m(L-x) + \left(\frac{h}{m\,k}\right)\sinh m(L-x)}{\cosh mL + \left(\frac{h}{m\,k}\right)\sinh mL}$	$M\,\dfrac{\sinh mL + \left(\frac{h}{m\,k}\right)\cosh mL}{\cosh mL + \left(\frac{h}{m\,k}\right)\sinh mL}$
Adiabatic tip: $\left(\frac{d\theta}{dx}\right)_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\left\lvert\dfrac{\cosh mL - (\theta_L/\theta_b)}{\sinh mL}\right\rvert$
Very Long fin (Infinite fin) ($x\to\infty$): $\theta(L) = 0$	e^{-mx}	M
$\theta = T - T_\infty$ $\theta_b = T_b - T_\infty$	$m = \sqrt{\frac{h\,P}{k\,A_c}}$ $M = \sqrt{hPkA_c}\,\theta_b$	

$$\textbf{Fin effectiveness,}\quad \varepsilon_f = \frac{q_f}{h\,A_{c,b}\theta_b}$$

$$\textbf{Fin efficiency,}\quad \eta_f = \frac{q_f}{h\,A_F\theta_b}$$

(2) External Flow (Fundamental of Convection)

Flow over a Flat Plate:

1.1 Laminar Flow

$$Velocity\ Boundary\ Layer\ thickness:\ \delta = \frac{5\,x}{\sqrt{Re_x}}$$

$$Thermal\ Boundary\ Layer\ thickness:\ \delta_t = \frac{\delta}{Pr^{1/3}} = \frac{5\,x}{Pr^{1/3}\sqrt{Re_x}}$$

$$The\ local\ friction\ coefficient\ (C_f)over\ a\ flat\ plate\ is:\ C_{f,x} = \frac{\hat{o}_s}{\tilde{n}\,u_\infty^2/2} = \frac{0.664}{\sqrt{Re_x}}$$

$$\frac{C_{f,x}}{2} = \left(\frac{h_x}{\tilde{n}Cp\,u_\infty}\right)Pr^{2/3} \qquad 0.6 < Pr < 60$$

1.2 Turbulent Flow

$$Velocity\ Boundary\ Layer\ thickness:\ \delta = \frac{\mathbf{0.37\,x}}{\mathbf{Re_x^{1/5}}}$$

The local friction coefficient (C_f) *over a flat plate is:*

$$C_{f,x} = \frac{\hat{o}_s}{\tilde{n}\, u_\infty^2/2} = \frac{0.0592}{Re_x^{1/5}} \qquad Re_{cr} \le Re \le 10^8$$

2. External forced Convection over Flat Plate:

The drag coefficient (friction coeff.) is: $C_D = F_D/\left(\frac{1}{2}\tilde{n}u_\infty^2 A\right)$

Where A is the frontal area of blunt bodies, and surface area for parallel flow over flat plate.

(a) **Smooth Surface**

$$Laminar: \ \overline{C_f} = 1.328\, Re_L^{-1/2} \qquad Re_L < 5x10^5$$

$$Turbulent: \ \overline{C_f} = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad 5x10^5 \le Re_L \le 10^7$$

(b) **Rough Surface Turbulent**

$$C_f = \left(1.89 - 1.62\, Log\frac{\mathring{a}}{L}\right)^{-2.5}$$

2.1 The Average Nusselt Number

$$Laminar: \ \overline{Nu} = 0.664\, Re_L^{0.5}\, Pr^{1/3} \qquad Re_L < 5x10^5$$

$$Turbulent: \ \overline{Nu} = (0.037\, Re_L^{0.8} - 871)\, Pr^{\frac{1}{3}} \qquad 5x10^5 \le Re_L \le 10^7 \ \& \ 0.6 \le Pr \le 60$$

3. Flow Over Cylinders and Spheres:

The average Nusselt number **over Cylinder**:

$$Nu_{cyl} = 0.3 + \frac{0.62\, Re^{0.5}\, Pr^{1/3}}{\left|1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right|^{1/4}} \left|1 + \left(\frac{Re}{282000}\right)^{5/8}\right|^{4/5} \qquad Re.Pr > 0.2$$

The average Nusselt number **over Spheres**:

$$\overline{Nu}_{sph} = 2 + \left|0.4\, Re^{\frac{1}{2}} + 0.06\, Re^{\frac{2}{3}}\right| Pr^{0.4} \left(\frac{\grave{\imath}_\infty}{\grave{\imath}_s}\right)^{\frac{1}{4}} \qquad 3.5 \le Re \le 80000$$

$$0.7 \le Pr \le 380$$

At Low Reynolds number

$$At\ Low\ Reynolds\ number: \ Nu_{D(\grave{e}=0)} = 1.15\, Re_D^{1/2}\, Pr^{1/3}$$

The average Nusselt number for cylinder is:

$$\overline{Nu}_D = \frac{\bar{h}\, D}{k} = C\, Re_D^m\, Pr^{1/3}$$

Re_D	C	m
0.4 – 4	0.989	0.330
4 – 40	0.911	0.385
40-4000	0.683	0.466
4000 – 40000	0.193	0.618
40000-400000	0.027	0.805

Flow Over Spheres:

The average Nusselt number is:

(3) Internal Forced Convection

$$Re < 2300 \quad Laminar\ flow$$

$$2300 \le Re \le 10000 \quad Transitional\ flow$$

$$Re > 10000 \quad Turbulent\ flow$$

Thermal Entrance Length (L_t):

Laminar: $L_{t,laminar} \approx 0.05\, Re\, Pr\, D$

Turbulent: $L_{t,turbulent} \approx L_{h,turbulent} \cong 10\, D$

Hydrodynamic Entrance Length (L_h):

Laminar: $L_{h,laminar} \approx 0.05\, Re\, D$

Turbulent: $L_{t,turbulent} \approx L_{h,turbulent} \cong 10\, D$

Pressure Drop and Pumping power

The pressure drop ΔP for internal flow in pipe is: $\Delta P = f\, \frac{L}{D}\left(\frac{\text{ñ} V_m^2}{2}\right)$; V_m the fluid mean velocity.

The pumping power $\dot{W}_{pump}$ for a tube bank is expressed as: $\dot{W}_{pump} = \dot{V}\, \text{Ä}P$

For turbulent Flow inside tube:

(a) **Smooth Tube**

$$f = (0.79 \ln Re - 1.64)^{-2} \qquad 10^4 < Re < 10^6$$

(b) Rough Tube (Fully developed flow)

$$\frac{1}{\sqrt{f}} = -2.0\, log\left(\frac{\text{å}/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \approx -1.8\, log\left[\frac{6.9}{Re} + \left(\frac{\text{å}/D}{3.7}\right)^{1.11}\right]$$

For Laminar Flow: In Entrance Region (L<L_t)

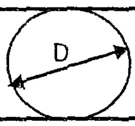
i) For circular tubes:

Fluid properties are evaluated at bulk mean fluid temperature.

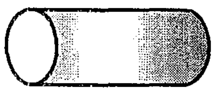
$$Nu = 3.66 + \frac{0.065(D/L)Re\, Pr}{1 + 0.04[(D/L)Re\, Pr]^{2/3}}$$

ii) For Fully Developed Region (L≥L_t):

Table ( ) Nusselt number and friction factor for fully developed laminar flow in tubes

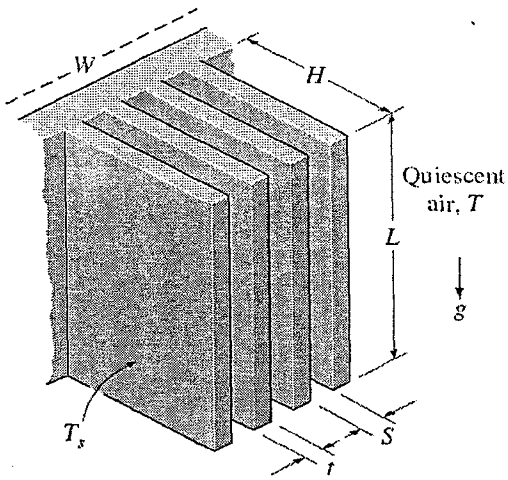
Tube Geometry	a/b or Θ	Friction factor, f	Nusselt number	
			T_s = const	$\dot{q}_s = const$
Circle 	-	$64.0/Re$	3.66	4.36

Free (Natural) Convection

Geometry	Recommended Correlation	Restrictions
Vertical Plate	$\overline{Nu}_L = \left\{0.825 + \dfrac{0.387\, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$	Non
Inclined plate (Cold surface up or Hot surface down)	$Ra_L = Gr_L Pr = \dfrac{(g\ cos\ \text{è})\text{â}(T_s - T_\infty)L^3}{\text{í á}}$ $\overline{Nu}_L = \left\{0.825 + \dfrac{0.387\, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$	$0 \le \theta \le 60°$
Horizontal plates a. Hot surface up or cold surface down.	$\overline{Nu}_L = 0.54\, Ra_L^{1/4}$ $\overline{Nu}_L = 0.15\, Ra_L^{1/3}$	$10^4 \le Ra_L \le 10^7$ $10^4 \le Ra_L \le 10^7$
Horizontal plates a. old surface up or hot surface down.	$\overline{Nu}_L = 0.27\, Ra_L^{1/4}$	$10^5 \le Ra_L \le 10^{10}$
Horizontal Cylinder 	$\overline{Nu}_L = \left\{0.60 + \dfrac{0.387\, Ra_L^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$	$Ra_D \le 10^{12}$
Sphere 	$\overline{Nu}_L = 2 + \dfrac{0.589\, Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$	$Ra_D \le 10^{11}$

Natural Convection from Extended Surface

$$Nu = \frac{hS}{k} = \left[\frac{576}{\left(Ra_s\, S/L\right)^2} + \frac{2.873}{\left(Ra_s\, S/L\right)^{0.5}}\right]^{-0.5}$$



Thermophysical Properties of Gases at Atmospheric Pressure

T, K	ρ, kg/m³	Cp (kJ/kg.K)	$\mu.10^7$ (N.m/s)	$\upsilon.10^6$ (m^2/s)	$k.10^3$ (W/*m.K)	$\alpha.10^6$ (m^2/s)	Pr
Air							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	3.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
Engine Oil (Unused)							
273	899.1	1.796	385	4280	147	0.910	47000
280	895.3	1.827	217	2430	144	0.880	27500
290	890.0	1.868	99.9	1120	145	0.872	12900
300	884.1	1.909	48.6	550	145	0.859	6400
310	877.9	1.951	25.3	288	145	0.847	3400
320	871.8	1.993	14.1	161	143	0.823	1965
330	865.8	2.035	8.36	96.6	141	0.800	1205
340	859.9	2.076	5.31	61.7	139	0.779	793
350	853.9	2.118	3.56	41.7	138	0.763	546
360	847.8	2.161	2.52	29.7	138	0.753	395
370	841.8	2.206	1.86	22.0	137	0.738	300
380	836.0	2.250	1.41	16.9	136	0.723	233
390	830.6	2.294	1.10	13.3	135	0.709	187
400	825.1	2.337	0.874	10.6	134	0.695	152
410	818.9	2.381	0.698	8.52	133	0.682	125
420	812.1	2.427	0.564	6.94	133	0.675	103
430	806.5	2.471	0.470	5.83	132	0.662	88
Saturated Water							
273.15	999.8	4.220	179.1	1.791	561.0	0.1330	13.47
275	999.9	4.214	168.2	1.682	564.5	0.1340	12.55
280	999.9	4.201	143.4	1.434	.574.0	0.1366	10.63
285	999.5	4.193	123.9	1.240	583.5	0.1392	8.91
290	998.8	4.187	108.4	1.085	592.7	0.1417	7.66
295	997.8	4.183	95.8	0.960	601.7	0.1442	6.66
300	996.5	4.181	85.4	0.8568	610.3	0.1465	5.85
305	995.0	4.180	76.7	0.7708	618.4	0.1487	5.18
310	993.3	4.179	69.4	0.6982	626.0	0.1508	4.63
320	989.3	4.181	57.7	0.5832	639.6	0.1546	3.77
340	979.5	4.189	42.2	0.4308	660.5	0.1610	2.68
360	967.4	4.202	32.6	0.3371	673.7	0.1657	2.03
373.15	958.3	4.216	28.2	0.2940	679.1	0.1681	1.75
400	937.5	4.256	21.9	0.2332	683.6	0.1713	1.36
420	919.9	4.299	18.7	0.2030	682.5	0.1726	1.18
440	900.5	4.357	16.3	0.1808	678.0	0.1728	1.05
460	879.5	4.433	14.4	0.1641	670.2	1.1719	0.955
480	856.5	4.533	13.0	0.1514	659.0	0.1697	0.892
500	831.3	4.664	11.8	0.1416	643.9	0.1660	0.853
520	803.6	4.838	10.8	0.1339	624.6	0.1607	0.833
540	772.8	5.077	9.9	0.1278	600.1	0.1530	0.835
560	738.0	5.423	9.1	0.1231	570.1	0.1425	0.864
580	697.6	5.969	8.3	0.1195	534.6	0.1284	0.931

Table (1) Nusselt number correlations for cross flow over tube banks of $N_L \geq 10$ and $0.7 < Pr < 500$

Arrangement	Range	Correlations
In-Line	0 – 100	$Nu_D = 0.9\, Re_D^{0.4} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$
	100 – 1000	$Nu_D = 0.52\, Re_D^{0.5} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$
	1000 – 2 x 10^5	$Nu_D = 0.27\, Re_D^{0.63} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$
	2 x 10^5 – 2 x 10^6	$Nu_D = 0.033\, Re_D^{0.8} Pr^{0.4} \left(\frac{Pr}{Pr_s}\right)^{1/4}$
Staggered	0 – 500	$Nu_D = 0.1.04\, Re_D^{0.4} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$
	500 – 1000	$Nu_D = 0.71\, Re_D^{0.5} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$
	1000 – 2 x 10^5	$Nu_D = 0.35 \left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.6} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$
	2 x 10^5 – 2 x 10^6	$Nu_D = 0.031 \left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.8} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$

Those relations can also be used for tube banks with N_L less than 16, but they are modified as:

$$\overline{Nu_D} = C_2 (\overline{Nu_D})_{N_L > 10}$$

Where F is correction factor whose values is given in Table(2).

$$Table(2) Correrection\ factor\ (F), for\ N_L < 16\ and\ Re_D > 1000$$

N_L	1	2	3	4	5	6	7	8	9
Aligned	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99
Staggered	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99

The exit temperature of fluid T_e from tube bundle is:

$$T_e = T_s - (T_S - T_i) exp\left(\frac{A_s h}{\dot{m} Cp}\right)$$

The mass flow rate $\dot{m}$ is:

$$\dot{m} = ñu_\infty (N_T S_T L)$$

The pressure drop ΔP for a tube bank is expressed as:

$$\Delta P = N_L f \div \left(\frac{ñV_{max}^2}{2}\right)$$

The pumping power $\dot{W}_{pump}$ for a tube bank is expressed as:

$$\dot{W}_{pump} = \dot{V}\, \Delta P =$$

Where ÷ is the correction factor and **f** is the friction factor given in **Figs(a & b).**

Flow across Tube Banks

The average Nusselt number is: (**$N_L \geq 10$**)

$$\overline{Nu}_D = C_1 Re_{D,max}^{m} \quad \begin{cases} N_L \geq 10 \quad 7 \quad Pr = 0.7 \\ 2000 \leq Re_{D,max} \leq 40000 \end{cases}$$

		S_T/D							
S_L/D		1.25		1.5		2.0		3.0	
		C_1	m	C_1	m	C_1	m	C_1	m
Aligned	1.25	0.348	0.592	0.275	0.608	0.100	0.704	0.0633	0.572
	1.50	0.367	0.586	0.250	0.620	0.101	0.702	0.0678	0.744
	2.00	0.418	0.570	0.299	0.602	0.229	0.632	0.1980	0.648
	3.00	0.290	0.601	0.357	0.584	0.374	0.581	0.2860	0.608
Staggered	0.60	-	-	-	-	-	-	0.213	0.636
	0.90	-	-	-	-	0.446	0.571	0.401	0.581
	1.00	-	-	0.497	0.558	-	-	-	-
	1.125	-	-	-	-	0.478	0.565	0.518	0.560
	1.250	0.518	0.556	0.505	0.554	0.519	0.556	0.522	0.562
	1.500	0.451	0.568	0.460	0.562	0.452	0.568	0.488	0.568
	2.000	0.404	0.572	0.416	0.568	0.482	0.556	0.449	0.570
	3.000	0.310	0.592	0.356	0.580	0.440	0.562	0.428	0.574

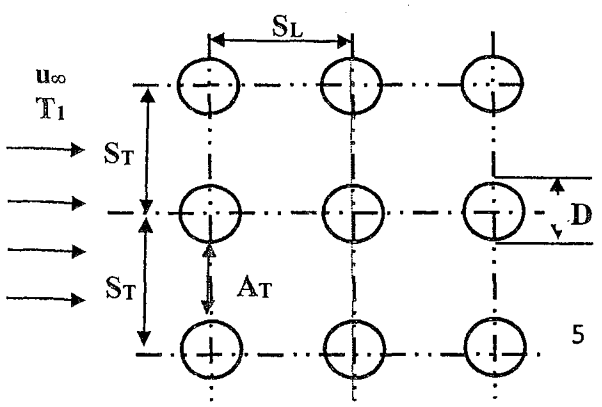
For Air: $\quad Re_D = \dfrac{V_{max} D}{\tilde{o}}$

For other fluids: $\quad Re_D = 1.13 \left(\dfrac{V_{max} D}{\tilde{o}} \right) Pr^{1/3}$

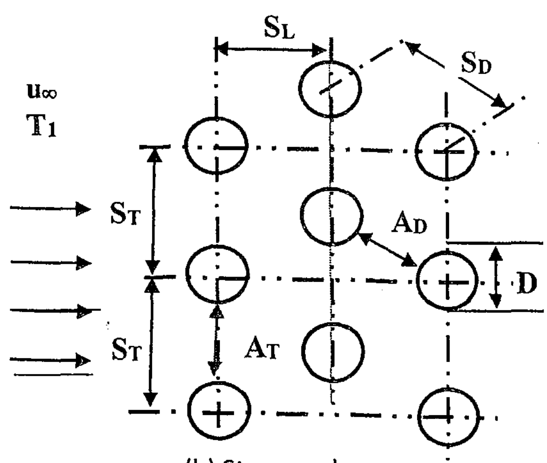
(I) **In-Line Arrangement** $\quad V_{max} = \left(\frac{S_T}{S_T - D} \right) u_\infty$

(II) **In-Staggered Arrangement with $S_D < \left(\frac{S_T + D}{2} \right)$** $\quad V_{max} = \left(\frac{S_T}{S_D - D} \right) u_\infty$

In-Staggered Arrangement with $S_D > \left(\frac{S_T + D}{2} \right)$ $\quad V_{max} = \left(\frac{S_T}{S_T - D} \right) u_\infty$



(a) In-Line



(b) Staggered

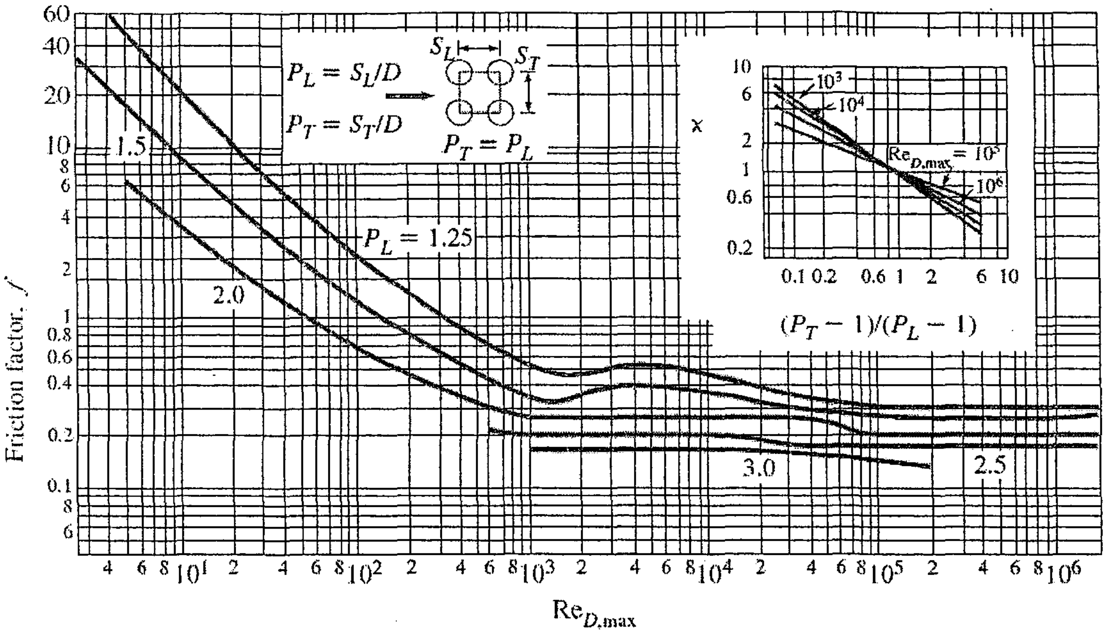


Fig.(a) Friction factor versus Reynolds number for in-line banks of tube.

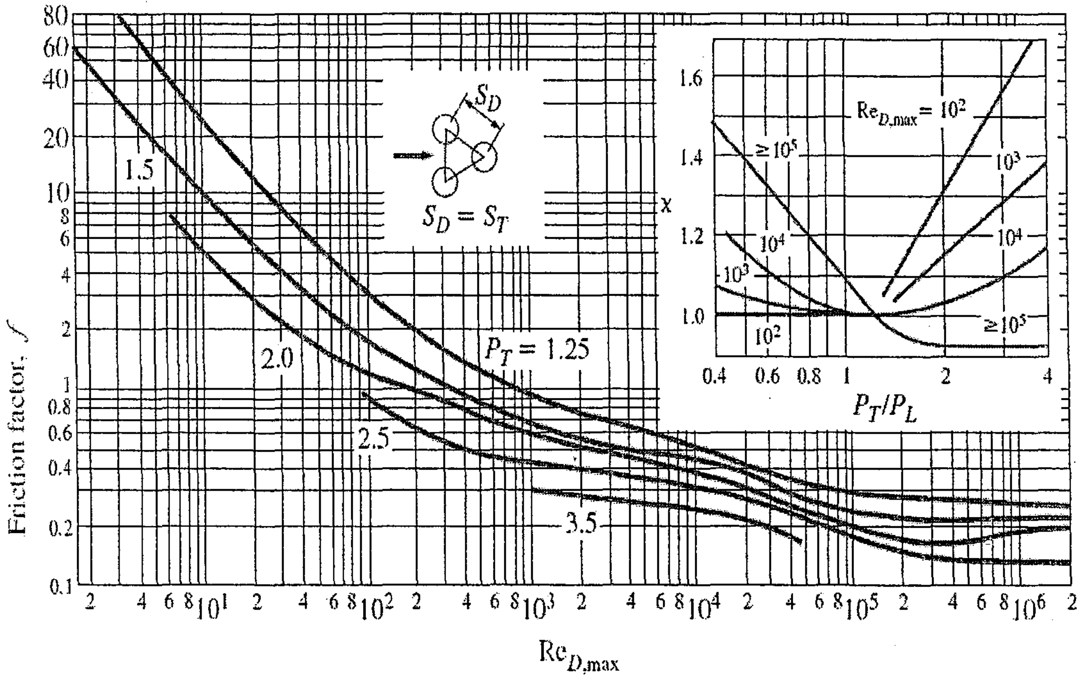


Fig.(b) Friction factor versus Reynolds number for staggered arrangement banks of tube.