

■ الامتحان أربع أسئلة في ست صفحات ■ أجب كل سؤال في المكان المخصص له ■ يفضل الحل بالقلم

Answer the following Problems

Problem 1

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- i) A dead cat was found within a closed room of a house where the temperature was a constant 25°C . At the time of discovery the core temperature of the cat was determined to be 33°C . **30 minutes** later a second measurement showed that the core temperature of the cat was 30°C . Assume that at the time of death the cat temperature was 38°C . **Determine** how many minutes elapsed before the dead cat was found?

Solution

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$f(x)$	e^x	a^x	$\sec x$	$\ln x$	$\sin x$	$\cosh x$	$\sinh x$	$\tanh x$	$\tan^{-1} x$	$\sec^{-1} x$	$\sin^{-1} x$
$f'(x)$	e^x	$a^x \ln a$	$\sec x \tan x$	$\frac{1}{x}$	$\cos x$	$\sinh x$	$\cosh x$	$\text{sech}^2 x$	$\frac{1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$	$\frac{1}{\sqrt{1-x^2}}$

ii) Test $z = (x - 2) \cosh x + y - e^y - \sinh x$ for maximum and minimum

5

Solution

$$z_x =$$

$$z_{xx} =$$

$$z_{yy} =$$

$$z_{xy} =$$

$$D =$$

$$z_y =$$

Critical Points

(,) Test the critical points (max., min., saddle, no decision) (,)

iii) If $w = f(x, y, z)$ is defined by $\sin(x^w) + y^{\sinh^{-1} x} + \ln\left(\frac{w^2 \sin^{-1} z}{e^y}\right) = 10$, find $\frac{\partial w}{\partial z}$

Solution

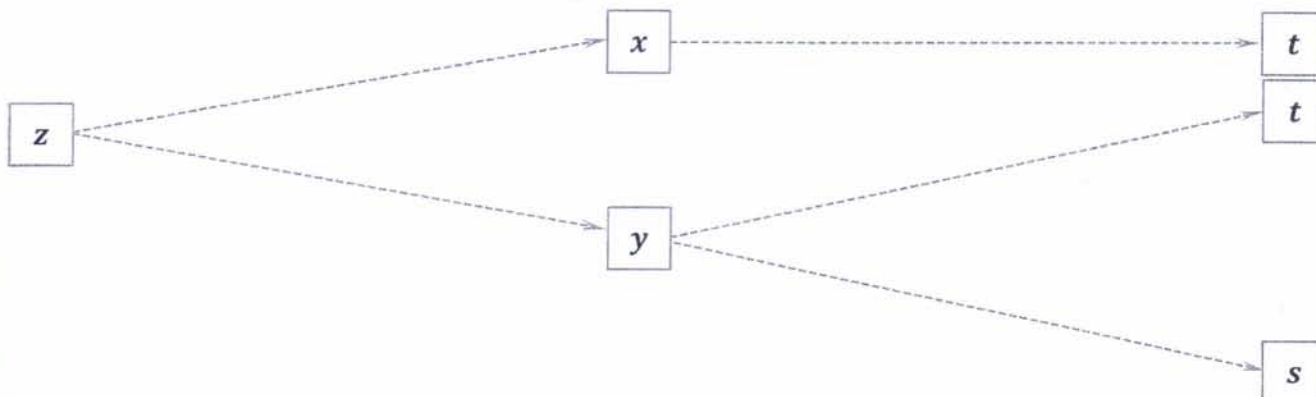
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Problem 2

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- i) Find $\frac{\partial z}{\partial s}$ for $z = y \tan^{-1}(x) + \ln(\sec y)$; $x = \sinh(t^t)$; and $y = t^s + \sec^{-1}(e^t)$

Solution

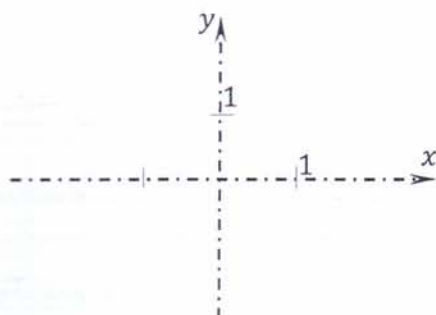
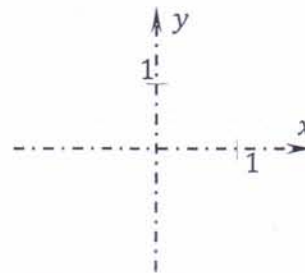


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- ii) Sketch the domain of $f(x, y) = \tan^{-1}(y - \tanh^{-1} x) + \ln(\operatorname{sech} x - y)$

Solution

5



Problem 3

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i) Find the value of $R = \lim_{x \rightarrow (-\infty)} (1 + \tanh x)^{\operatorname{sech} x}$

Solution

5

ii) If $x = 3$ is a root of multiplicity two of $f(x) = x^5 - 6x^4 + 9x^3 + ax + b$

A) Find a and b (using the synthetic division) B) Sketch the graph of $f(x)$.

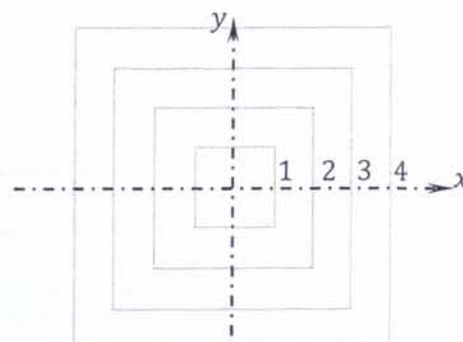
Solution

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$\therefore a = \quad, b =$

Root: $x =$	Type (root of multiplicity?)
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Problem 4

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i) If $\lambda = 5$ is an eigen value of $A = \begin{pmatrix} k & 1 \\ 2 & 3 \end{pmatrix}$, Find the value of k .

Solution

ii) For the system, $x + 2y = k$, $x + hy = 1$. Determine the value(s) of h and k such that the system has: i) No solution ii) Unique solution iii) Many solution.

Solution

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$$\Rightarrow \left[\begin{array}{c|c} & \end{array} \right] \quad \left[\begin{array}{c|c} & \end{array} \right]$$

i) No solution:
if

ii) Unique solution:
if

if iii) Many solutions:
if

iii) Use partial fractions to decompose $F = \frac{3x^2 + x + 1}{x^3 + x}$

Solution

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Finally $\frac{3x^2 + x + 1}{x^3 + x} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$

(5/6)

- i) Find the inverse of A
(using Gauss Jordan elimination method)

where $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ 2 & 3 & 1 \end{pmatrix}$

Solution

1	1	1	1	0	0
-1	-1	0	0	1	0
2	3	1	0	0	1

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$$A^{-1} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

- ii) Using A ,
solve the following system

$$\begin{aligned} x + y + z &= 1 \\ x + y &= -1 \\ 2x + 3y + z &= 1 \end{aligned}$$

Solution

$$\begin{pmatrix} & & \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$