



**Question 1: [10 Marks] complete the following:**

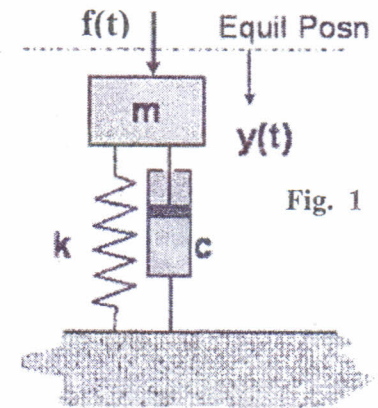
(i) The order of the ODE  $(y'')^4 + (y'')^3 - y^5 = 0$  is .....while the degree is.....

(ii) If  $Lf(t) = F(s)$ , then  $L[t e^{3t} f(t)] = \dots\dots\dots$

(iii) If D is the circle  $x^2 + y^2 = 25$ , then the value of  $\iint_D 4 \, dA = \dots\dots\dots$

(iv) The divergence of the vector field  $\vec{V}(x, y, z) = (yz^2)i + (xyz)j + (y \sin x)k$  is.....while its curl is.....

(v) If  $L^{-1} F(s) = f(t)$ , then  $L^{-1} F'(s) = \dots\dots\dots$

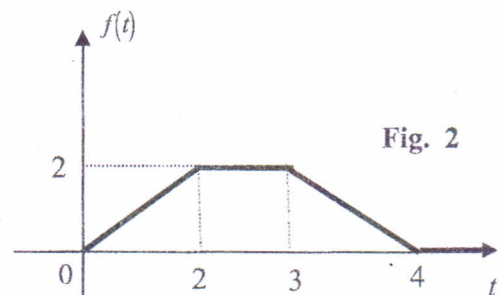


**Question 2 [13 Marks]**

(a) Find the orthogonal trajectory to the family of curves  $y = ce^{-x^2}$ .

(b) Find the solution to the IPV  $y' + 2y = \sqrt{y}$ ,  $y(0) = 0$ .

(c) Find the general solution to the ODE  $x^2 y'' + xy' = \ln x$ .



**Question 3 [12 Marks]**

(a) Find Laplace transform to the signal shown in Fig. 2.

(b) Evaluate the improper integral:  $\int_0^\infty \frac{\sin(2t)}{t} dt$ .

(c) In spring mass system, in Fig. 1, Find: (i)  $y(t)$  if  $m = k = 1$ ,  $c = 0$  and  $f(t) = \delta(t - \pi) + \delta(t - 2\pi)$

(ii) The force  $f(t)$  that produces  $y(t) = \sin(t)$ , assume zero initial velocity and zero initial displacement.

**Question 4 [15 Marks]**

(a) Find the local maximum and the local minimum of the function:  $f(x, y) = x^2 + y^2 - 2x - 2y + 3$ .

(b) Find the rate of change of the surface:  $(x - 2)^2 + (y + 3)^2 + (z - 1)^2 = 25$  at a point  $(2, 2, 1)$  in the direction,  $i + 2j + k$ . Find also the maximum rate of change at that point.

(c) Find: (i)  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ .

(ii)  $\int_0^2 \int_{(x/2)}^1 \cosh(y^2) dy dx$

Best of luck,

Prof. Ibrahim L. El-Kalla