Mansoura University
Faculty of Engineering

Fall Semester Exam.
Level 200
Wednesday 2/1/2018

Biomedical Engineering Programs

Prof. Dr. Magdi S. El-Azab Numerical Analysis (MTH 201) Time allowed: 2 Hours.

Answer the following questions (Full mark 50 marks). (All computations have to be executed to four decimal places)

Question 1 [24 marks]

(a) [8 marks] By the least square technique, derive the normal equations to fit the curve $y = ax + b$ to the data $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$. Given the table of readings:

x	0.5	1	1.5	2	2.5
y	15.08	8.32	5.96	4.74	3.99

Fit the curve $y = \left(\frac{a + \sqrt{x}}{b\sqrt{x}}\right)^2$, with the computations of $L_2 - norm\ of$ the error to the above readings.

(b) [8 marks] By the use of two different interpolation formulas approximate $y(0.75)$ and $y(1.6)$.

(c) [8 marks]. Find the positive two roots of the algebraic equation

$$x^4 - 12x^2 + 35.3 = 0,$$

correct to five decimal places by two different methods, one method for each root. Guess the other two roots.

باقى الاسئلة فى الصفحة التالية

Question 2 [27 marks]

(a) [9 marks] Given the initial value problem:

$$y' = \sqrt{10x^2 + y}, \quad x_0 = 0.2, \quad y_0 = 0.41,$$

(i) Evaluate $y(0.4)$ by Runge-Kutta approximation of order 4,

(ii) Find $y(0.7)$ by the use of Taylor method,

(iii) Find $y(0.8)$ by the use of Euler method.

(b) [9 marks] Starting with the initial values

$$\left(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, x_4^{(0)}\right) = (2, 2, -2, -2),$$

solve the following system of equations correct to three decimal places

$$\begin{aligned} 2x_1 + 0.5x_2 + 0.5x_3 + x_4 &= 6 \\ x_1 + 4x_2 - x_3 + x_4 &= 8 \\ 2x_1 - x_2 + 4x_3 - 0.5x_4 &= 5 \\ 2x_1 + x_2 + 3x_3 + 5x_4 &= 2 \end{aligned}$$

(c) [9 marks] Given the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 12(x + y), \quad 0 < x < 2, \quad 0 < y < 2$$

Take the mesh sizes $h = k = 0.5$ and the unknown u is given to be $4(4x^2 + y)$ on the boundaries. Apply the finite difference method and the relaxation method to solve this problem with the initial values

$$\begin{aligned} u_{11}^{(0)} &= 4, \quad u_{21}^{(0)} = 6, \quad u_{31}^{(0)} = 8, \\ u_{12}^{(0)} &= 10, \quad u_{22}^{(0)} = 12, \quad u_{32}^{(0)} = 14, \\ u_{13}^{(0)} &= 16, \quad u_{23}^{(0)} = 18, \quad u_{33}^{(0)} = 20 \end{aligned}$$

With all best wishes