



Mansoura University  
Faculty of Engineering

Final Examination  
Self-study  
Monday 26/8/2019  
Open Book Exam.

Biomedical Engineering Programs



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Mathematics 4 (MTH 102)

Time allowed: 2 Hours.

**Answer the following questions (Full mark 50 pts).**

**Question 1 [25 marks]**

1. (a) [8 pts] Classify the points  $x = 0$ , and  $x = -1$  for the differential equation

$$4xy'' + 2y' + xy = 0,$$

and then solve it in series about  $x = 0$ .

(b) [8 pts] Evaluate each of the following integrals

$$(i) I_1 = \int_{-\pi/2}^{\pi/2} \sin^2(x) \cos^3 dx,$$

$$(ii) I_2 = \int_0^1 \left( \frac{1}{\sqrt{-\ln x}} \right)^5 dx$$

(c) [9 pts] Prove that,

(i) the system  $\left\{ 1, x, \frac{3}{2}x^2 - \frac{1}{2} \right\}$ , is orthogonal over the interval  $[-1, 1]$ , hence normalize it.

(ii)  $\Gamma(x + 1) = x\Gamma(x)$ .

(iii) If  $f(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ , show that

$$\int [f(x + 2) - f(x)] dx = \ln x(x + 1) + C.$$

باقى الاسئلة فى الصفحة التالية

**Question 2 [25 marks]**

(a) [5 pts] Classify the partial differential equation

$$xu_{xx} + 2(y-1)u_{xy} + u_{yy} = 0.$$

Use graphs in the  $xy$  -plane to explain your answer.

(b) [10 pts] Find Fourier integrals of the function

$$f(x) = \begin{cases} \cos x & -\pi \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

(c) [10 pts] Use the technique of separation of variables to solve the following boundary value problem

$$u_{xx} + u_{yy} = 0, \quad 0 < x < \pi, \quad 0 < y < \pi$$

$$u(0, y) = 0, \quad 0 < y < \pi$$

$$u(\pi, 0) = 0, \quad 0 < y < \pi$$

$$u_y(x, 0) = 0, \quad 0 < x < \pi$$

$$u(x, \pi) = x^3, \quad 0 < x < \pi$$

With all best wishes