Mansoura University Faculty of Engineering Mechanical Power Eng. Dept. Course Title: 3D Modelling of Fluid Flow and Heat Transfer Course Code: MPE371		Academic Programs: BSc Biomedical Engineering Exam Date: 4/1/2020 Exam Type: Final Exam Time: 2 hours Full Mark: 50

Attempt to answer all questions. Assume any missing data.

Question 1 [24 marks]

The following equation describes the transport of the variable ϕ

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla.\left(\rho\phi\vec{U}\right) = \nabla.(\Gamma\nabla\phi) + S_\phi$$

The problem parameters are given in the table below

Parameter	Value
Inflow boundary condition	5
outflow boundary condition	10
Density	1
Γ	0.1
Inlet velocity	2.2

i. Simplify the equation to model a one-dimensional source-free steady convection-diffusion in the x-direction. **[2 marks]**

ii. Using the finite volume method, convert the model to a system of algebraic equations applying appropriate discretisation schemes. Use the variables ϕ_0 and ϕ_L to express Dirichlet boundary conditions at the inlet and outlet, respectively. Assume uniform cross sectional area. Illustrate your derivation with relevant sketches. **[14 marks]**

iii. Write the system of algebraic equations for a uniform mesh consisting of 5 volumes. What is the type of the coefficients matrix? **[2 marks]**

iv. Using the Jacobi-iterative method, perform two iterations towards solving the system of algebraic equations using appropriate initial guess. **[4 marks]**

v. Sketch the results of each iteration in one graph. **[2 marks]**

Question 2 **[12 marks]**

The governing equations of compressible fluid flow are:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{U}) = 0 \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t} + \nabla.(\rho u\vec{U}) = -\frac{\partial p}{\partial x} + \nabla.(\mu\nabla u) + [s_{Mx}] + \rho g_x \tag{2}$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla.(\rho v\vec{U}) = -\frac{\partial p}{\partial y} + \nabla.(\mu\nabla v) + [s_{My}] + \rho g_y \tag{3}$$

$$\frac{\partial(\rho w)}{\partial t} + \nabla.(\rho w\vec{U}) = -\frac{\partial p}{\partial z} + \nabla.(\mu\nabla w) + [s_{Mz}] + \rho g_z \tag{4}$$

$$\frac{\partial(C_v\rho T)}{\partial t} + \nabla.(C_v\rho T\vec{U}) = -p\nabla.\vec{U} + \nabla.(k\nabla T) + \Phi + S_i \tag{5}$$

i. Starting from the conservation of mass principle; derive the continuity equation. Illustrate your derivation with sketches. **[5 marks]**

ii. Starting from the conservation of mass principle, Newton's second law of motion, and the continuity equation; derive the left-hand side of the momentum equations. **[2 marks]**

iii. Derive the governing equations for steady incompressible flow in 2D using equations 1, 2, 3 and 4. Neglect body forces. Expand any divergence or gradient terms. Assume a constant coefficient of viscosity μ. **[5 marks]**

Use the following definitions:

$$[s_{Mx}] = \left[\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x} + \lambda\nabla.\vec{U}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial x}\right)\right]$$

$$[s_{My}] = \left[\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y} + \lambda\nabla.\vec{U}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial y}\right)\right]$$

$$[s_{Mz}] = \left[\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial z}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial z}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z} + \lambda\nabla.\vec{U}\right)\right]$$

Question 3 [8 marks]

Mention four differences between solving the steady incompressible Navier-Stokes equations and solving the steady convection-diffusion equation for a general variable ϕ when the velocity field is known? Discuss how these differences are dealt with in CFD codes.

Question 4 [6 marks]

Suppose the transport equation of the variable ψ in the 2D space coordinates x and y is given by:

$$a\frac{\partial\psi}{\partial x}+b\frac{\partial\psi}{\partial y}=c\frac{\partial^2\psi}{\partial x^2}+d\frac{\partial^2\psi}{\partial y^2}$$

where a, b, c and d are known constants.

What is the type of the equation and its effect on the solution domain for the following cases:

i. $a=0$, $b=0$, $c=1$, and $d=1$

ii. $a\neq 0$, $b\neq 0$, $c=0$, and $d=0$

iii. $a\neq 0$, $b\neq 0$, $c=0$, and $d\neq 0$

Illustrate your answer with relevant sketches

Best Wishes
Dr Elsafei Zidan and Dr Yahia Fouda