Course Name: NanoTechnology
(Nanophotonics)
Course Code: MPE471
Midterm Exam
BME Program
Level 400
Allowed Time: 60 Minutes

Attempt all questions. Assume any missed data. Full mark is 25.

Q.1 [5 Marks] Answer the following questions:

1. [2 Marks] Define Phase velocity and group velocity of an electromagnetic (light) wave. Is the group velocity ever greater than the phase velocity?

2. [2 Marks] Explain with the aid of net sketches the wavefronts of an electromagnetic plane wave and that of a spherical wave. What is the relation between the directions of electric, magnetic fields and the direction of propagation of an electromagnetic wave?

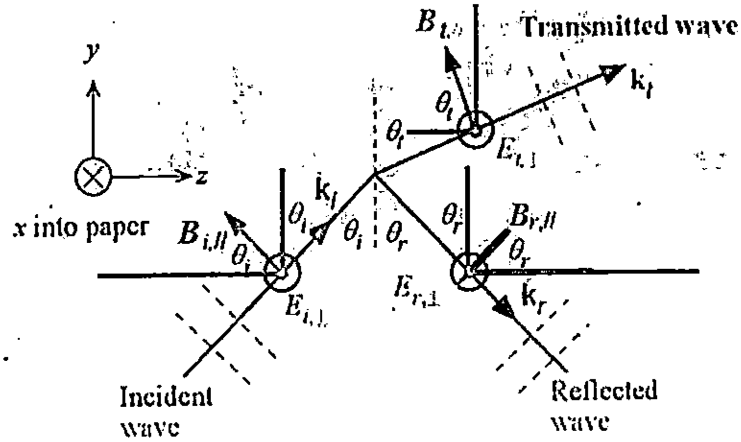
3. [1 Marks] Explain briefly Fermat's principle of least time.

Q.2 [8 Marks] Using Fresnel Equations and according to the below figure, prove that the reflection coefficient ($r_\perp$) and transmission coefficient ($t_\perp$) of the normal electric field components are given by:

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

Where: $n = \frac{n_2}{n_1}$



Q.3 [5 Marks] A light beam traveling in air is incident on a glass plate of refractive index 1.5. What is the Brewster or polarization angle? What are the relative intensities of the reflected and transmitted light for the polarization perpendicular and parallel to the plane of incidence at the Brewster angle of incidence?

Q.4 [7 Marks] For a single layer antireflection (AR) coating of index n_2 on a material with index n_3 ($> n_2 > n_1$), as shown in Figure 1.57(a), the minimum reflectance at normal incidence is given by:

$$R_{min} = \left[\frac{n_2^2 - n_1 n_3}{n_2^2 + n_1 n_3}\right]^2 \qquad \textit{Single layer AR coating}$$

However, the choice of materials may not always be the best for a single layer AR coating. Therefore, double layer AR coatings can be used alternatively to achieve lower and sharper reflectance at a specified wavelength as shown in Figure 1.57(b) and (c). The reflections A, B and C for normal incidence result in a minimum reflectance given by:

$$R_{min} = \left[\frac{n_3^2 n_1 - n_4 n_2^2}{n_3^2 n_1 + n_4 n_2^2}\right]^2 \qquad \textit{Double layer AR coating}$$

Double layer reflectance vs. wavelength behavior usually has V-shape, and they are called V-coatings as may be seen in Figure 1.57(c).

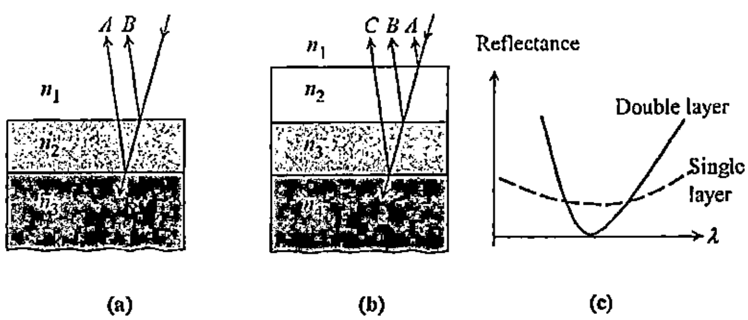


Figure 1.57(a) A single layer AR coating. (b) A double layer AR coating and its V-shaped reflectance spectrum over a wavelength range.

(a) Show that double layer reflectance vanishes when $\left[\frac{n_2}{n_3}\right]^2 = \frac{n_1}{n_4}$ (Best double layer AR coating).

(b) Consider an InGaAs, a semiconductor crystal with an index 3.8, for use in a photodetector. What is the reflectance without any AR coating?

(c) What is the reflectance when InGaAs is coated with a thin AR layer of Si_3N_4? Which material in the table would be ideal as an AR coating?

TABLE 1.3 **Typical AR materials and their approximate refractive indices over the visible wavelengths**

	MgF_2	SiO_2	Al_2O_3	CeF_3	Sb_2O_3	Si_3N_4	SiO	ZrO_2	ZnS	TiO_2	CdS
n	1.38	1.46	1.65	1.65	1.9–2.1	1.95	2.0	2.05	2.35	2.35	2.60

(d) What two materials would you choose from the above table to obtain a V-coating between air and InGaAs substrate?

My best wishes to all of you!
Prof. Salah Obayya

Formula Sheet

$$r_{//} = \frac{E_{r0,//}}{E_{i0,//}} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_{//} = \frac{E_{t0,//}}{E_{i0,//}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$R_{\perp} = \frac{\left|E_{ro,\perp}\right|^2}{\left|E_{io,\perp}\right|^2} = \left|r_{\perp}\right|^2$$

$$R_{//} = \frac{\left|E_{ro,//}\right|^2}{\left|E_{io,//}\right|^2} = \left|r_{//}\right|^2$$

$$T_{\perp} = \frac{n_2\cos\theta_t\left|E_{to,\perp}\right|^2}{n_1\cos\theta_i\left|E_{io,\perp}\right|^2} = \left(\frac{n_2\cos\theta_t}{n_1\cos\theta_i}\right)\left|t_{\perp}\right|^2$$

$$T_{//} = \frac{n_2\cos\theta_t\left|E_{to,//}\right|^2}{n_1\cos\theta_i\left|E_{io,//}\right|^2} = \left(\frac{n_2\cos\theta_t}{n_1\cos\theta_i}\right)\left|t_{//}\right|^2$$